

Proceedings of the Workshop on  
The Use and Calibration  
of the  
Kern ME5000 Mekometer

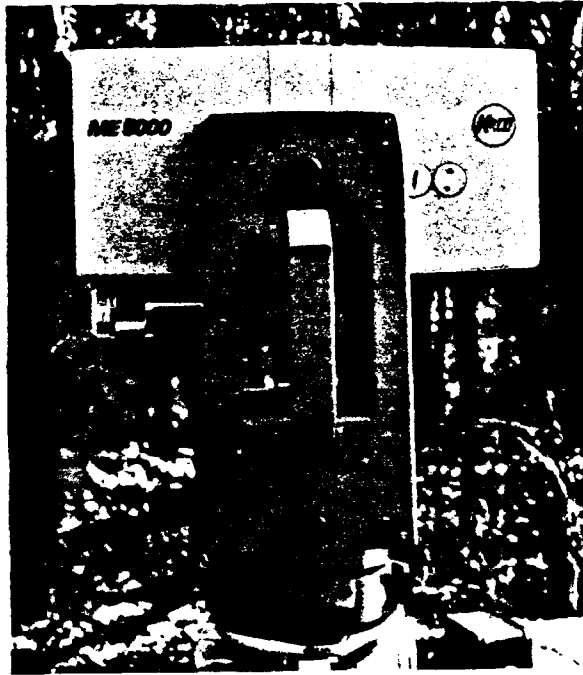


June 18-19, 1992

Prepared for the Department of Energy  
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STANFORD LINEAR ACCELERATOR CENTER  
Stanford University • Stanford, California

# Proceedings of the Workshop on The Use and Calibration of the Kern ME5000 Mekometer



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*Stanford Linear Accelerator Center  
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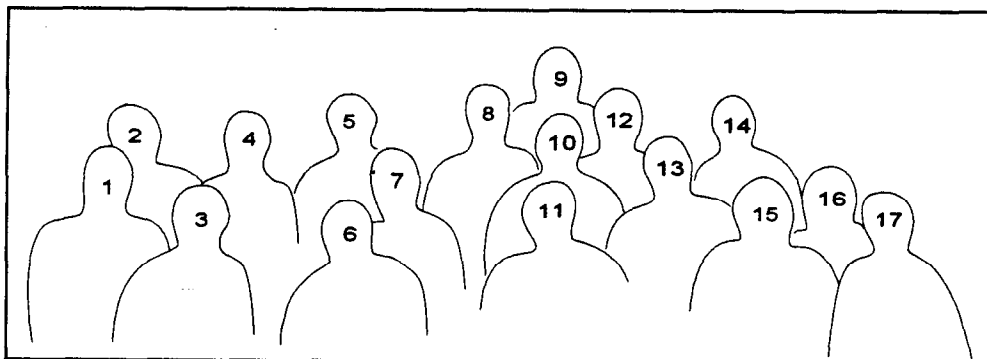
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## INTRODUCTION

When it was introduced in 1986 the Kern ME5000 Mekometer immediately attracted attention from the survey groups of accelerator laboratories around the world. This interest was generated not only because the ME5000 offered higher accuracy than any other distance measuring instrument on the market, but also because, unlike previous instruments, its measuring range extended down to just a few meters, thus opening up the whole realm of accelerator housings to high-accuracy EDM measurements. Although Kern quoted a range of 20-8000 m, software was quickly made available to extend the measurement range below 20 m (the PROMEKO program developed by the Technical University of Munich).

In 1988-89 three DOE laboratories (CEBAF, FNAL and LBL) acquired instruments. Since none of these laboratories had equipment to calibrate these instruments whereas SLAC did, extensive tests were made with all three instruments at SLAC in 1989.<sup>1</sup> When four other laboratories initiated the procurement process in the second half of 1991 to buy their own machines, plans were set in motion to make a full series of test measurements with all DOE instruments, bringing them to SLAC one at a time. Measurements commenced in January 1992 when the LBL, FNAL and CEBAF instruments were measured in quick succession. The ANL and BNL instruments, both new, followed in February and March. In June SLAC received its own instrument and measurements began right away.

As what was to have been the culmination of this project, a workshop was held at SLAC in mid-June to discuss the results. 16 people from 7 DOE laboratories attended as well as one representative from the Pohang Light Source in Korea (see the list of participants on p. iv). By this time measurements were complete for five instruments and mostly complete for a sixth (the SLAC instrument). This volume is the formal record of the six papers that were presented during the two days of the workshop. The first paper covers ME5000 Operation - how the instrument works. The second describes the measurements that were made at SLAC with each instrument. Data analysis is described in the third and fourth papers, and the results are presented in the fifth paper. The final paper is a special invited paper commissioned from Chris Curtis of CEBAF who spent three years (1987-89) working with a ME5000 on the Channel Tunnel project in England.

The Workshop proved not to be the culmination of the project for two more instruments were measured in August - the two SSC instruments. The workshop papers presented in this volume have been updated to incorporate these results. The baseline measurements for the SLAC instrument have not been completed and sufficient time has elapsed that they must be started afresh, something that is not feasible in the next few months. These updated proceedings therefore represent the culmination of this project to test the ME5000 instruments at use in each DOE laboratory.

SLAC  
September 22, 1992

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<sup>1</sup> T.W. Copeland-Davis, Can the Kern ME5000 Mekometer Replace Invar Measurements? Results of Test Measurements with Three Machines, *Proceedings of the First International Workshop on Accelerator Alignment*, SLAC, July 31 - August 2, 1989, pp. 171-183.

# ME5000 OPERATION

Bernard Bell, SLAC

## 1. BASIC PRICIPLES OF EDM

The basic principle behind all EDM instruments is the same: a beam of known frequency  $F$  is sent to a target and reflected back to the instrument. This round-trip distance  $2D$  includes an integral number of wavelengths  $m\lambda$  plus a fractional part of a wavelength  $f\lambda$ ,

$$2D = m\lambda + f\lambda \quad (1)$$

The desired result is the one-way distance  $D$  not the round-trip distance  $2D$ . It is usual to describe this one-way distance  $D$  as containing the same number  $m$  of *half-wavelengths*  $\lambda/2$ ,

$$D = m\frac{\lambda}{2} + f\frac{\lambda}{2} \quad (2)$$

The precise value of this half-wavelength  $\lambda/2$ , also known as the *effective wavelength*, depends upon the refractive index of the air through which the beam travels,  $n_{\text{ri}}$

$$\frac{\lambda}{2} = \frac{c_0 n_{\text{ri}}}{2F} \quad (3)$$

in which  $c_0$  is the velocity of light *in vacuo*,  $299792458 \text{ ms}^{-1}$ .

Three methods have been used to find  $m$  and  $f$  and thus find the distance  $D$ .

### 1.1. Phase Resolution

Most infra-red EDM instruments, such as the DM503, determine the fractional part  $f$  by measuring the phase angle  $\Delta\phi$  between the outgoing beam and the return beam, (4) and Fig. 1.

$$D = m\frac{\lambda}{2} + \frac{\Delta\phi}{2\pi} \frac{\lambda}{2} \quad (4)$$

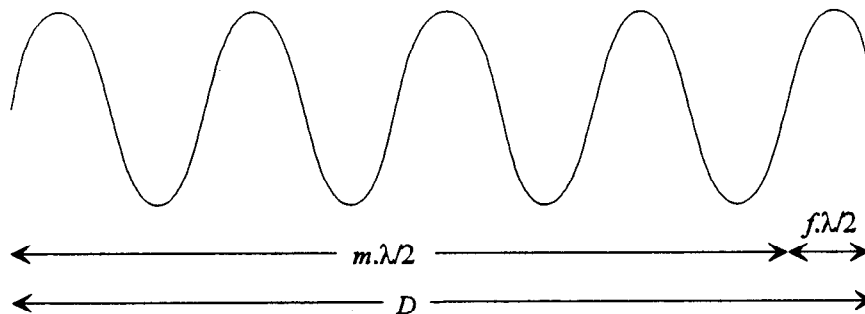


FIGURE 1. Distance determination by phase resolution

Such an approach, though the cheapest and simplest, introduces cyclic error into the measurements. To find the value of  $m$ , the measurement is repeated using several predefined frequencies.

## 1.2. Path Length Modification

The Kern ME3000 Mekometer eliminates the fractional part  $f$  by changing the path distance  $D$  until the phase difference is zero, indicating an integral number of half-wavelengths, (5) and Fig. 2.

$$D + \delta D = m \frac{\lambda}{2} \quad (5)$$

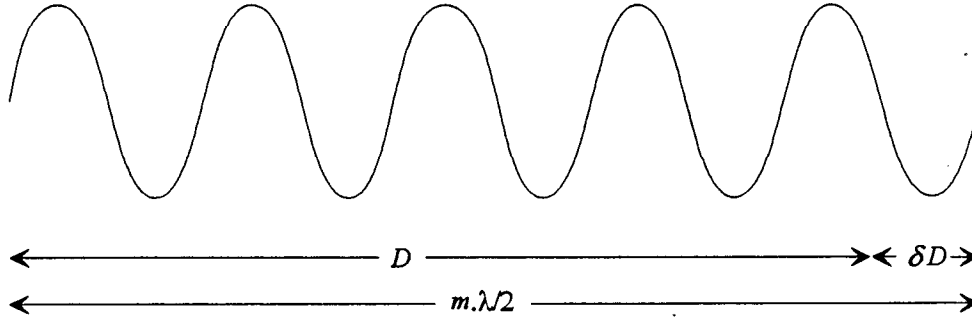


FIGURE 2. Distance determination by changing the path length

As in the first approach,  $m$  is found by repeating the measurement using several predefined frequencies.

## 1.3. Frequency Modification

The Kern ME5000 Mekometer also eliminates the fractional part  $f$ , but by using a different method. The frequency is changed until a zero phase difference is obtained, (6) and Fig. 3.

$$D = m \frac{\lambda}{2} \quad (6)$$

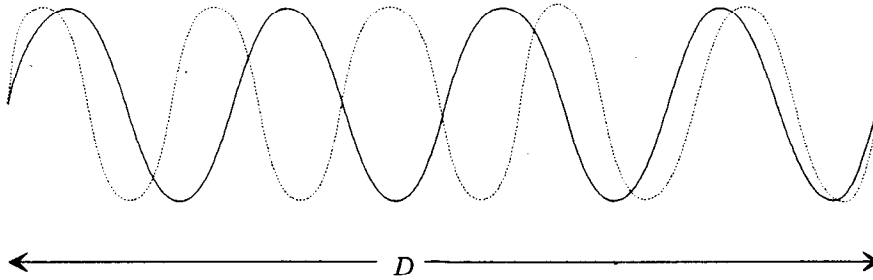


FIGURE 3. Distance determination by changing the frequency

The value of  $m$  is found in a manner different from the first two approaches. The frequency is increased in small steps until a zero phase difference is again detected, at which frequency the path contains  $m+1$  of these new half-wavelengths  $\lambda_1/2$ ,

$$D = (m+1) \frac{\lambda_1}{2} \quad (7)$$

These two frequencies provide sufficient information to resolve  $m$  and thus the distance.

The second and third approaches afford the ME3000 and ME5000 greater accuracy for two reasons:

- a) The null condition or zero point at which there is no phase difference can be located with greater accuracy than a definite phase difference can be measured.
- b) Since the phase difference is always the same there is no cyclic error.

## 2. THE MODULATION SIGNAL

The carrier beam is a HeNe laser (Class II, 1 mW) of 632.8 nm wavelength. Onto this is superimposed the reference signal which is polarization modulated at a frequency of  $c.500$  MHz, giving an effective wavelength ( $\lambda/2 = c/2F$ ) of 30 cm. The nominal range of the modulator crystal is 460 - 510 MHz. Within this range the modulation bandwidth of the modulation is a nominal 15 MHz. The quartz oscillator from which the modulation signal is derived is compensated for temperature effects to ensure high accuracy.

Before a measurement can be made the instrument must perform a modulator calibration using function SF141. The power intensity output of the modulator is measured at 75 intervals throughout the entire nominal range 460 - 510 MHz. From this calibration the upper and lower limits of the *normal* (nominal 15 MHz) and *extended* (nominal 30 MHz) bandwidths are determined.

The plot of power intensity against frequency is termed the *characteristic curve*. Fig. 4 shows the nominal characteristic curve as shown in the Kern literature. The actual measured curves can differ substantially from this nominal shape. Fig. 5 shows the measured curves for two instruments. The reason for the discrepancy in shapes is not known.

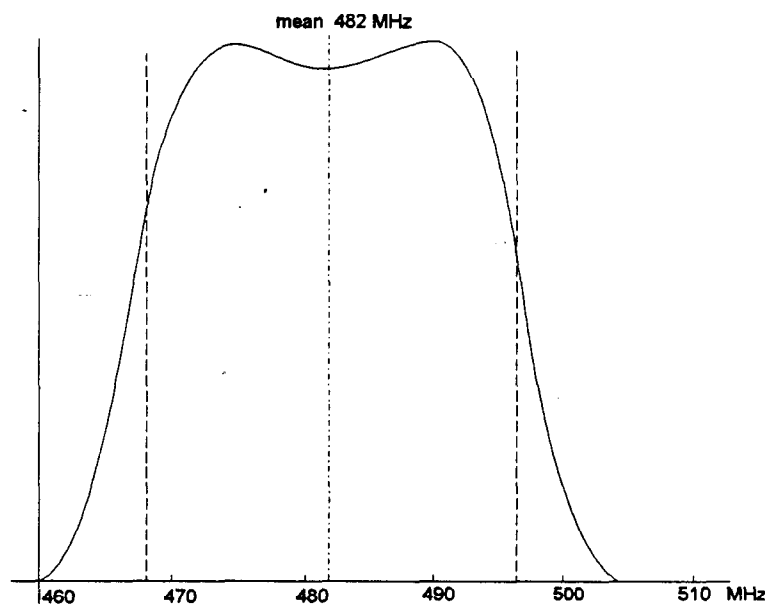


Figure 4. Nominal Characteristic Curve

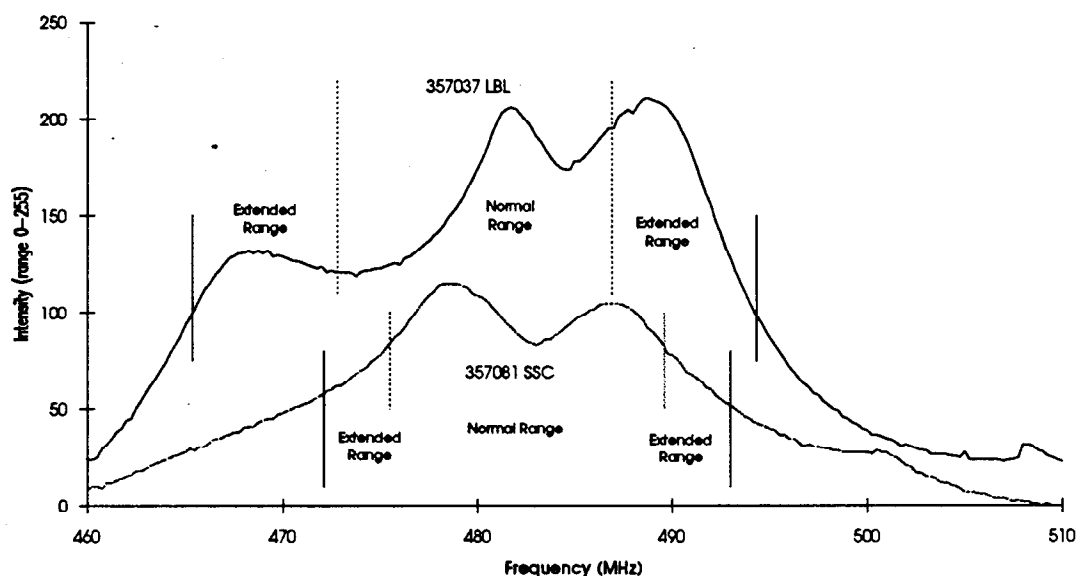


Figure 5. Actual Characteristic Curves for Instruments 357037 and 357081

For short-distance measurements, below 20 m, the normal 15 MHz bandwidth is insufficient (for reasons explained below), so the intensity threshold is reduced to 75% of maximum to give a wider bandwidth, nominally 30 MHz. In practice, this bandwidth is dependent upon the characteristics of the individual crystal, and has been found to vary in the range 18 - 29 MHz. Table 1 shows the normal and extended bandwidths for the two instruments shown in Fig. 5.

Table 1. Modulation bandwidth of instruments 357037 and 357081

	NORMAL		EXTENDED	
	Range (MHz)	Bandwidth	Range (MHz)	Bandwidth
357037 (LBL)	472.112 - 486.915	14.803	465.383 - 494.644	28.261
357081 (SSC)	472.785 - 486.915	14.130	469.420 - 490.280	20.859

### 3. RESOLVING THE DISTANCE

#### 3.1 Normal Mode

SLAC's field measurement and data collection program ME5000.BAS mimics the operation of the onboard program in the ME5000's EPROM for the measurement of distances in the normal operating range of the instrument, 20 - 8000 m.

##### 3.1.1. Calculation of distance from frequency

Starting at the lower end of the bandwidth an upward sweep is made until a frequency  $F_1$  is found at which the phase difference is zero. This *nodal point* occurs when there is an integer number  $m_1$  of half-wavelengths along the measurement path. The sweep is continued until a second nodal point is found at frequency  $F_{1A}$ , at which there are  $m_1+1$  half-wavelengths. The distance can be determined from the frequency difference  $F_{1A}-F_1$ , (8-10).

$$m_1 = \frac{2 F_1 n_{ri}}{c_o} D \quad (8)$$

$$m_1 + 1 = \frac{2 F_{1A} n_{ri}}{c_o} D \quad (9)$$

$$D = \frac{c_o}{2 n_{ri} (F_{1A} - F_1)} \quad (10)$$

The frequency difference  $F_{1A} - F_1$  is proportional to the distance, Table 2.

Table 2. Correlation between frequency difference and distance

Distance m	$F_{1A} - F_1$ MHz
5	29.971
10	14.985
20	7.493
50	2.997
100	1.499
1000	0.150
8000	0.019

The range limitations of the ME5000 can be understood from Table 2. Kern specifies a working range of 20 - 8000 m. Within this range all distances can be measured unambiguously in stand-alone mode using the software programmed into the EPROM in the instrument. The upper limit of 8 km is imposed by the minimum step size in the coarse and fine search routines. The lower limit of 20 m is imposed by the requirement of finding two adjacent nodal points within the bandwidth of 15 MHz. Distances above 8 km and below 20 m can be measured only using special software.

If possible, more than two nodal points are used in order to improve the accuracy. Up to four nodal points are located, Fig. 6.

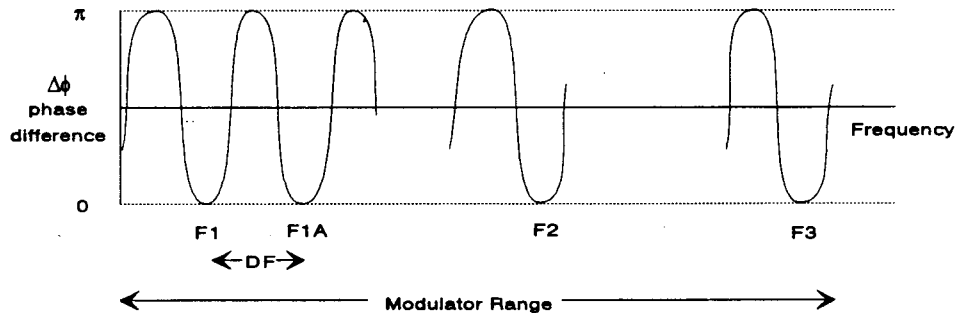


Figure 6. Location of null points

1.  $F_1$ , the first nodal point found in the upward sweep from the lower end of the bandwidth.
2.  $F_{1A}$ , the nodal point adjacent to  $F_1$ . The approximate locations of  $F_2$  and  $F_3$  are then calculated using the frequency interval between nodes,  $F_{1A} - F_1$ .
3.  $F_2$ , the nodal point approximately in the center of the bandwidth
4.  $F_3$ , the nodal point nearest the upper limit of the bandwidth.

If they exist,  $F_1$ ,  $F_2$ , and  $F_3$  are used for the final distance calculation. If only three frequencies are

found,  $F_2 = F_{1A}$ . For a few distances in the range 20-25 m only two frequencies,  $F_1$  and  $F_3$ , are found.

At each nodal point the frequency is determined by a coarse measurement followed by a fine measurement.

### 3.1.2. Coarse search in normal mode

Using the onboard coarse search function (SF144) the frequency is increased by predefined increments, starting from the lower end of the bandwidth, until a zero point (zero phase difference) is found. Each zero point is indicated by a reduction in the light intensity falling on the detector diode in the instrument. When the outgoing and return signals are exactly in phase no light falls on the diode, Fig. 7.

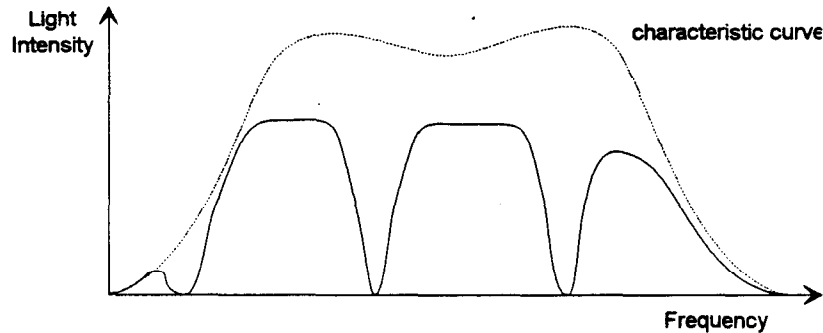


FIGURE 7. Light intensity at the receiver diode

### 3.1.3. Fine search in normal mode

The onboard function SF145 performs the fine search routine. Due to background noise it is difficult to pick up the minimum signal itself. Instead, the instrument superimposes a 2 kHz wobbler frequency. The amplitude of this signal is  $\pm 6.7$  kHz for high range ( $>500$  m) and  $\pm 35$  kHz for low range (20 - 500 m). When the oscillation of this 2 kHz signal covers the zero point itself it produces a 4 kHz signal. This means that the coarse search must locate the zero point to within half the bandwidth of this wobbler frequency. If the mean frequency does not exactly coincide with the zero point a 2 kHz signal is also produced, Fig. 8(a). When the mean frequency does coincide with the zero point this 2 kHz signal disappears, Fig. 8(b). The wobbler frequency is adjusted in steps of 162 Hz until the zero point is located. This smallest step of 162 Hz is equivalent to a change in wavelength of  $0.1 \mu\text{m}$ .

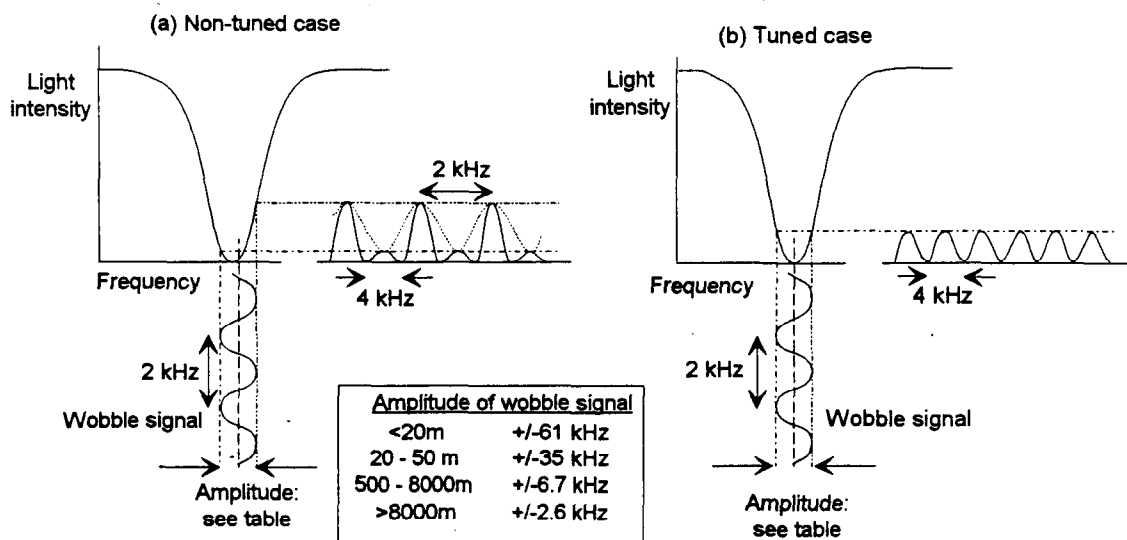


FIGURE 8. Determination of the zero point using a 2 kHz wobbler signal

The precise frequency of the zero point is the weighted average of five sets of 256 individual measurements (a total of 1280 individual determinations of the zero point).

### 3.2. Short Distance Measurements

In stand-alone mode using its onboard program, measurements can be made only in the normal operating range 20 - 8000 m. For measurement above 8 km or below 20 m, an external computer must guide the instrument through the necessary measurement steps. The SLAC program ME5000.BAS incorporates the necessary routines for measurements below 20 m, but not for measurement above 8 km as the need has never arisen.

For distances below 20 m a different search method is used, as described below.

#### 3.2.1 Coarse search in short-distance mode

Rather than use the coarse search function (SF144) in the ME5000  $\mu$ P, a coarse search is made under the control of the ME5000 program. In response to code 68, the ME5000 returns the phase position  $\phi$  in hexadecimal in the range 00 - FF. 7F indicates zero phase, but is ambiguous: it indicates a phase difference of either 0 or  $\pi$ . On earlier EDM instruments with a manual phase meter (e.g., ME3000), whether the signals are in phase or exactly out of phase is determined by the direction in which the needle is moving. Similarly, in the ME5000 the two signals are in phase if the phase decreases with decreasing frequency or increases with increasing frequency for slight changes in frequency around the null point. Fig. 9 shows the phase signal across the entire extended bandwidth for a distance of 20m. There are four nodes, two in the normal bandwidth and two in the extended bandwidth. The transition from  $\phi = 00$  to  $\phi = FF$  covers just a few kHz.

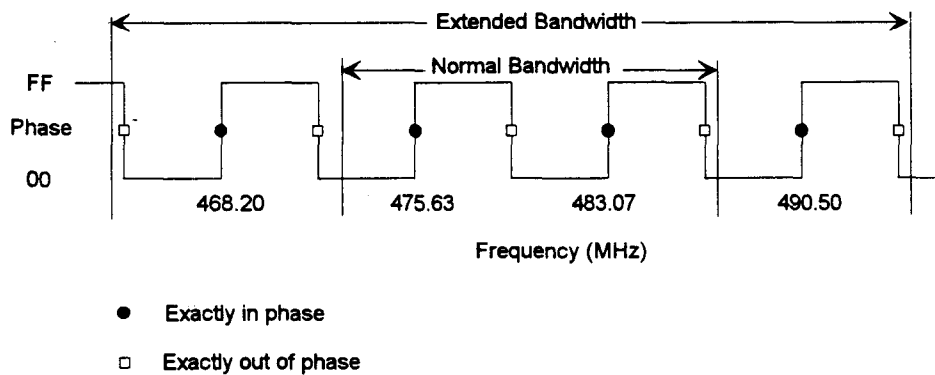


FIGURE 9. Phase signal across the extended bandwidth for a 20m distance

Using increasingly fine intervals (2, 0.25, 0.0625, 0.03125, 0.005176 MHz) the program searches back and forth for the upper and lower edges of the window around the zero point, as indicated by the transition from  $\phi = 00$  to  $\phi = FF$ . The midpoint between these upper and lower edges is taken as the coarse value for the zero point and is used as the starting value for the fine search.

#### 3.2.2. Fine search in short-distance mode

The fine search is then made using the onboard fine search function (SF144), as for normal range, but with the amplitude of the wobble signal set to  $\pm 61$  kHz.

If only one frequency is found, an approximate distance is required, so that the node number can be calculated, (11-13).

$$\lambda_1 = \frac{c_o}{2 n_{ri} F_1} \quad \begin{array}{l} \lambda_1 = \text{wavelength for frequency } F_1 \\ D_{apx} = \text{approx. distance} \end{array} \quad (11)$$

$$m_1 = \text{rnd} \left( \frac{D_{apx} + ADC}{\lambda_1} \right) \quad \begin{array}{l} ADC = \text{addition constant} \end{array} \quad (12)$$

$$D = m_1 \lambda_1 - ADC \quad (13)$$

Short distance measurements below 20 m form the bulk of the work performed with the ME5000 at particle accelerators. The majority of these distances are already known to within a millimeter or so. The ME5000 program provides a feature to enter a pre-known distance. This distance must be sufficiently close to the actual distance that the wobbler frequency falls over the zero point during the coarse search (see Fig. 8 and description). The entry of a valid preknown distance produces a considerable saving in time.

# ME5000 TEST MEASUREMENTS

Bernard Bell, SLAC

## 1. ERRORS IN EDM INSTRUMENTS

Calibration of EDM instruments is necessary in order to determine three different types of error: offset error, cyclic error and scale error.

### 1.1. Offset error

The largest error and the one that must always be determined prior to any measurement is what is known variously as the *zero offset*, the *offset error*, or the *instrument constant*. This is a constant error caused by the inability to perfectly align the optical and electronic components to the mechanical center of the instrument. Fortunately, as well as being the largest error, the offset error is the easiest to determine. The simplest determination can be made by measuring all three distances between three colinear points (Fig. 1).

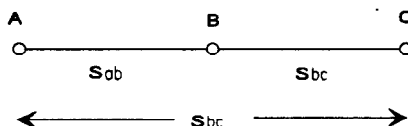


FIGURE 1. Simple method for determining the zero offset

Each of the three distance measurements ( $S_{AB}$ ,  $S_{AC}$ ,  $S_{BC}$ ) contains the true distance plus the zero offset,  $\epsilon_o$ . The offset is therefore easily calculated,

$$\epsilon_o = S_{AC} - (S_{AB} + S_{BC}) \quad (1)$$

The addition constant of each ME5000 is determined in Switzerland prior to shipping the unit. The value of the offset is set on rotary switches in the instrument. The value set on these switches is not the actual value but that value subtracted from 20 cm. It is set to an accuracy of 0.1 mm.

Table 1. ME5000 Addition constants

	Serial number	Switch settings cm	Addition constant m
LBL	357037	5.81	0.1419
Fermilab	357046	5.78	0.1422
CEBAF	357036	5.77	0.1423
Argonne	357086	5.74	0.1426
Brookhaven	357088	5.84	0.1416
SLAC	357089	5.80	0.1420

### 1.2. Cyclic error

Cyclic error affects only those instruments that use phase resolution - this includes most of the infrared instruments such as the DM503, but not the ME5000. Since the ME5000 always locates a constant zero phase difference it is immune to cyclic error.

### 1.3. Scale error

Scale error is due to an error in the frequency of modulation of the carrier beam. Determination of the error therefore consists in measuring the emitted frequency and comparing this against the frequency the instrument thinks it is using. The method used to do this is described below in Section 4 on Frequency Measurements.

Three series of test measurements were conducted in order to evaluate the behavior of each instrument: baseline measurements, interferometer measurements and frequency measurements. These tests are described below.

## 2. BASELINE MEASUREMENTS

### 2.1. The SLAC Baseline

The SLAC baseline was constructed in mid-1985 along the south side of the Linac. Twenty pillars (B01 - B20) were erected (Table 2). The baseline runs nearly due east-west from B01 in the west at Sector 0-0 above the Linac injector to B20 in the east at Sector 21-2.

Each pillar is a 1-ft diameter reinforced concrete post, poured *in situ*, that protrudes 3 ft above ground and descends 4 ft beneath ground. Each pillar is topped with a CERN socket. Most of the pillars are on the grass verge beside the access road.

The linac slopes downward from the west end at a slope of approximately 0.005 (0.3°). The intention was to set out the baseline in such a manner that the CERN sockets atop the pillars follow a geometrical straight line rather than the earth's curved surface, and that this line have the same slope as the linac. The elevation for each socket was calculated by applying a curvature correction working outwards from B02, the pillar closest to the midpoint of the line, and using a slope of 0.00498382. However, this design was not finally achieved. Since it was found that the design location of B02 would place the pillar in a parking area, B02 was actually installed 10 m closer to B03. New curvature corrections were not calculated, but the resultant error is minor (see Table 6).

TABLE 2. The spacing of pillars on the SLAC baseline

Pillar	Distance from B01 (m)	Distance to next pillar (m)
B01	-	950
B02	950	530
B03	1480	30
B04	1510	20
B05	1530	40
B06	1570	50
B07	1620	50
B08	1670	50
B09	1720	50
B10	1770	20
B11	1790	50
B12	1840	40
B13	1880	50
B14	1930	30
B15	1960	30
B16	1990	50
B17	2040	50
B18	2090	10
B19	2100	40
B20	2140	-

Of the twenty pillars, only eleven are used for EDM calibrations (Table 3). The spacing of these pillars is such as to provide a broad range of distances with no duplication (Table 4).

TABLE 3. The pillars used for EDM baseline calibrations

B01												
B02	950											
B03	1480	530										
B05	1530	580	50									
B10	1770	820	290	240								
B12	1840	890	360	310	70							
B14	1930	980	450	400	160	90						
B15	1960	1010	480	430	190	120	30					
B17	2040	1090	560	510	270	200	110	80				
B19	2100	1150	620	570	330	260	170	140	60			
B20	2140	1190	660	610	370	300	210	180	100	40		
	B01	B02	B03	B05	B10	B12	B14	B15	B17	B19	B20	

TABLE 4. The distances measured in EDM baseline calibrations

From	To	Distance	From	To	Distance	From	To	Distance
B14	B15	30	B10	B17	270	B03	B20	660
B19	B20	40	B03	B10	290	B02	B10	820
B03	B05	50	B12	B20	300	B02	B12	890
B17	B19	60	B05	B12	310	B01	B02	950
B10	B12	70	B10	B19	330	B02	B14	980
B15	B17	80	B03	B12	360	B02	B15	1010
B12	B14	90	B10	B20	370	B02	B17	1090
B17	B20	100	B05	B14	400	B02	B19	1150
B14	B17	110	B05	B15	430	B02	B20	1190
B12	B15	120	B03	B14	450	B01	B03	1480
B15	B19	140	B03	B15	480	B01	B05	1530
B10	B14	160	B05	B17	510	B01	B10	1770
B14	B19	170	B02	B03	530	B01	B12	1840
B15	B20	180	B03	B17	560	B01	B14	1930
B10	B15	190	B05	B19	570	B01	B15	1960
B12	B17	200	B02	B05	580	B01	B17	2040
B14	B20	210	B05	B20	610	B01	B19	2100
B05	B10	240	B03	B19	620	B01	B20	2140
B12	B19	260						

The other nine pillars of the baseline were included to allow distnvar measurement of most of the baseline. 50 m is the longest practical distance that can be measured with invar wire and the distnvar. With these nine auxilliary pillars, the 660 m from B03 to B20 can be measured with 10, 20, 30, 40 and 50 meter wires. This ability to measure the baseline with invar wires was important for detecting any scale difference between the invar wires used for measuring distances in the accelerator housings and the DM503 used for measuring the geodetic network on the surface.

For two reasons this comparison against invar wires is not necessary for the ME5000. Firstly, the ME5000 can be used both on the surface network and in the tunnels. Secondly, the scale of the ME5000 can be determined much more easily by means of a frequency calibration.

Located alongside a busy access road the baseline has received some damage in its seven years. One of the distnvar pillars, B07, was destroyed by a garbage truck in late 1985. Because this pillar was on a paved road rather than on the grass verge like most of the other pillars, it was not rebuilt. Instead, a concrete pad was poured, flush with the road surface. This pad has three holes drilled so a tunnel tripod can be mounted and a CERN socket plumbed over a target set in the concrete. More seriously, one of the EDM pillars, B20, received a major blow from a Pettibone forklift that moved the pillar about 5 cm.

## 2.2. Preparation for Measurements

Before the first ME5000 was calibrated in January 1992, the baseline was prepared for measurements. Overhanging branches (fast-growing eucalyptus) were cleared to provide clear lines of sight. Each CERN socket on the 11 baseline pillars was carefully levelled.

Direction measurements were made using an E2 theodolite to determine the horizontal offsets of the eleven pillars. Table 5 shows the offsets from the line through B01 and B19. These results clearly show the misalignment of B20. It is only on the shortest distance, B20-B19, that this 55 mm misalignment of B20 has a significant effect: 38  $\mu$ m over 40 m. The effect on all other distances is negligible.

Table 5. Horizontal offsets of the baseline pillars

	Distance (m)	Offset (m)
B01		0.00000
B02	950	-0.05247
B03	1480	-0.01663
B05	1530	-0.01319
B10	1770	-0.03595
B12	1840	-0.00297
B14	1930	-0.00652
B15	1960	-0.02726
B17	2040	0.01472
B19	2100	0.00000
B20	2140	0.05551

Elevations of the CERN sockets were determined by running a level line along the linac using a Wild N3 level. The vertical offsets from the best-fit straight (not ellipsoidal) line are shown in Table 6. These offsets are negligible.

Table 6. Vertical offsets of the baseline pillars

	Elevation (m)	Offset (m)
B01	102.79639	0.02416
B02	98.06176	-0.05665
B03	95.42034	0.01164
B05	95.17114	0.00572
B10	93.97503	0.00222
B12	93.62616	0.00592
B14	93.17762	0.00546
B15	93.02810	0.00333
B17	92.62940	0.00208
B19	92.33037	-0.00037
B20	92.13101	-0.00351
$\sigma_n$		0.01921
slope		-0.004973

## 2.3. ME5000 Measurements

All measurements are made at night to reduce atmospheric effects. This is especially important at SLAC because the baseline is built alongside a paved road which is in use during the day but quiet at night, and because of the large temperature swings during a typical day in this part of California.

A complete set of measurements of all eleven pillars includes 55 distances. It is usual to make two complete sets of measurements, one in each direction. This is done either by measuring two independent sets, first in a westerly direction then in an easterly direction, preferably on different nights, or by combining the two sets by measuring all ten distances from each pillar. The latter method was used for all the 1992 measurements. Between the forward and reverse measurements at each station the instrument

was leveled in order to provide two quasi-independent sets of measurements.

Since a complete set of ten distances measured from one pillar with a ME5000 Mekometer takes about 1.5 hours using SLAC's measurement software (the BASIC program ME5000), 3-4 nights are required for a complete set of bidirectional of measurements.

The measurements were reduced to the arc distance on SLAC's reference surface: the ellipsoid (Clarke's) at station 100 (77.6437 m) at the east end of the linac. The two sets of data (forward and back) were analyzed separately using Koch's method of variance components.<sup>1</sup> This program solves for three unknowns: the addition constant and the two-part accuracy statement. The accuracy specification of an EDM instrument  $\sigma$  is usually broken down into a constant error  $\sigma_K$  independent of distance, and a proportional error  $\sigma_S$  dependent on distance. Koch's method determines these two components of the variance,

$$\sigma^2 = \sigma_K^2 + \sigma_S^2 S^{2H} \quad (2)$$

S is the distance, and the value of H is chosen according to whether accuracy increases with distance ( $H > 0$ ) or decreases with distance ( $H < 0$ ). It has been found from experience that the most suitable alternative values for H are  $-1/2$ ,  $+1/2$ , and 1.

### 3. INTERFEROMETER MEASUREMENTS

#### 3.1. The Calibration Laboratory

The Survey calibration laboratory is an underground chamber approximately 5 x 30 m, located at Sector 10 of the linac and on the same level as the accelerator. The main feature of the room is a 30 m interferometer bench. The HP laser interferometer on this bench serves as the length standard for the survey and alignment work at SLAC. Against this standard are calibrated invar wires, level rods, and such distance measuring instruments as can measure below 30 m.

#### 3.2. ME5000 Measurements

Two series of measurements were undertaken with each instrument.

##### 3.2.1. Resolution Test

This test involved moving the reflector over a distance of 1-2 mm in steps of 100  $\mu$ m in order to determine the resolution of the instrument. The ME5000 was mounted at the near end of the bench, some 35 m from the laser head. Two standard HP interferometer reflectors were mounted back to back in a block which was placed on the bench near the laser head. One reflector faced the laser, the other faced the ME5000, 30 m distant. The block was mounted on a carriage attached to a linear stepping motor. Under computer control, this carriage was moved along the bench in increments of  $\approx 100 \mu$ m. At each stop, note was made of the distance as measured by the interferometer and as measured by the ME5000.

##### 3.2.2. Accuracy Test

The same block, housing back-to-back reflectors was placed on a carriage that could be rolled the full length of the bench. Starting near the laser head, 30 m from the instrument, the carriage was moved in 1m steps toward the ME5000. The interferometer and ME5000 readings were recorded for each stop.

The ME5000 measurements were corrected for refractive index before comparison with the interferometer readings.

<sup>1</sup> Hans Fröhlich, Die Bestimmung der äußeren Genauigkeit elektrooptisch gemessener Strecken mittels Varianzkomponentenschätzung.

#### 4. FREQUENCY MEASUREMENTS

Since the accuracy of any EDM instrument is at least partly dependent upon the accurate knowledge of the modulation frequency, it is important to ensure that there is no significant frequency drift over time. The basic equation for determining the distance is shown in (3). The distance  $D$  equals a whole number  $m$  of half-wavelengths  $\lambda/2$ , plus some fraction  $f$  of an additional half-wavelength.

$$D = m \frac{\lambda}{2} + f \frac{\lambda}{2} \quad (3)$$

Three different methods are used in EDM instruments in order to find  $f$ , the fractional part of a half-wavelength.

1. Infra-red instruments, such as the DM503, use a fixed frequency and measure the phase difference between the reference signal and the returning beam. Such an approach, though the simplest and cheapest, introduces cyclic error.
2. The frequency of the ME3000 Mekometer is allowed to vary as changing atmospheric conditions change the dimensions of the reference cavity. A system of prisms is moved within the instrument so as to increase the path length until the fractional component  $a$  is eliminated. This approach eliminates cyclic error.
3. The ME5000 Mekometer varies the frequency until there is an integral number of wavelengths in the distance. Like the ME3000, this approach eliminates cyclic error.

The frequency calibration of the ME5000 is more straightforward than that of the earlier ME3000 Mekometer. Because the carrier wave of the ME5000, a HeNe laser ( $\lambda = 632.8$  nm), is continuously modulated the modulation frequency can be directly measured by any frequency counter capable of operating at 500 MHz (Fig. 2). Since there is no reference cavity in the ME5000, the modulation frequency is not dependent on the ambient temperature and pressure as is the ME3000.

A SMA jack, similar to that on the ME3000, is located on the underside of the ME5000. A SMA-BNC adapter is included with the instrument so that a standard coaxial cable can be connected to this outlet. The function switch should be set to either Battery or Remote. Set to Battery, the frequency currently set is shown on the display. Set to Remote, any frequency within the range of the instrument (460 - 510 MHz) can be set externally.

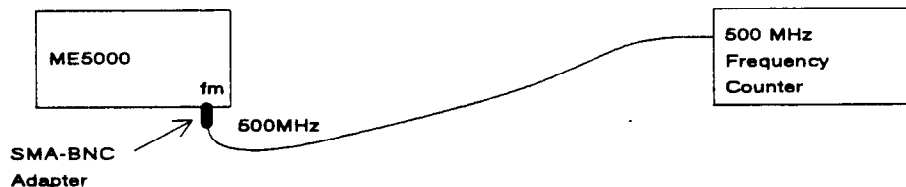


FIGURE 2. Schematic for ME5000 frequency calibration

At SLAC we use a HP 5342A microwave frequency counter in one of the Klystron test labs. It is calibrated annually and is reliable to a few tens of Hertz. As long as power is supplied to the counter, the crystal is kept at a stable temperature even when the main power switch is off.

## 4.1. MEKTEST

A small BASIC program MEKTEST is used to perform the frequency calibration. This program provides several useful pieces of information. Following each description below is the relevant portion of the MEKTEST output file.

### 4.1.1. Instrument settings

The program reads and records the ASB address and the zero offset set on the switches inside the instrument. This zero offset value (5.80 cm in the example below) is not the actual addition constant; that is obtained by subtracting the switch setting from 20 cm (giving 0.1420 m in the example below).

```
ME5000 Mekometer Calibration
06-10-1992  14:17:50
Serial Number : 357089
Belonging to  : SLAC

ASB # : 1
VERSION : ME5000 / 1 / 22.06.88
CONSTANT : 5.8 cm
```

### 4.1.2. Modulator calibration

Using the built-in function SF141, the ME5000 performs a calibration of its modulator, measuring the output of the modulator at 75 equally spaced frequency points in the range 460 - 510 MHz. This is done in order to determine the optimum bandwidth limits in both normal and extended modes.

```
MODULATION FREQUENCY RANGES (MHz)
06-10-1992  14:18:22
      Lower      Upper      Range
Standard  473.458  487.588  14.130
Extended  466.056  494.990  28.934
```

### 4.1.3. Frequency calibration

For a frequency calibration the ME5000 must be connected to a frequency counter. The instrument is instructed to step through its frequency range with an interval selectable by the user. At each step the program records the frequency set on the instrument and the operator is prompted to enter the frequency recorded on the counter. The default parameters are to step from 466 MHz to 492 MHz in steps of 2 MHz. In the sample output below the discrepancy between the counter and the ME5000 is consistently 20 Hz. Since this represents only 0.04 ppm, and is within the noise level of the counter, there is no significant frequency error.

```
FREQUENCY CALIBRATION
06-10-1992  14:21:02
Nominal      Set          Counter      Diff
466  465.999943  465.999915  +28
468  467.999975  467.999957  +18
470  470.000007  469.999985  +22
472  472.000040  472.000020  +20
474  474.000072  474.000055  +17
476  475.999943  475.999921  +22
478  477.999975  477.999957  +18
480  480.000008  479.999991  +17
482  482.000040  482.000019  +21
484  484.000072  484.000054  +18
486  485.999943  485.999923  +20
488  487.999976  487.999957  +19
490  490.000008  489.999992  +16
492  492.000040  492.000020  +20
```

#### 4.1.4. Characteristic curve

The characteristic curve is the plot of signal output versus frequency over the range of the modulator (460 - 510 MHz). This is the curve that is measured by the ME5000 when a modulator calibration is performed. MEKTEST performs a similar measurement but under its own control and with a smaller interval. The output of the modulator is measured from 460 - 510 MHz in steps of 0.25 MHz, a total of 201 measurements. The modulator output is obtained in hexadecimal (range 00 - FF) and converted to decimal (range 0 - 255). Figure 3 shows the characteristic curve and the bandwidth limits for the SLAC instrument.

CHARACTERISTIC CURVE  
06-30-1992 11:40:09

Freq.	Hex	Dec
460.00	15	21
460.25	17	23
460.50	19	25
460.75	1B	27
461.00	1F	31
461.25	21	33
461.50	23	35
461.75	26	38
462.00	28	40
:	:	:
:	:	:

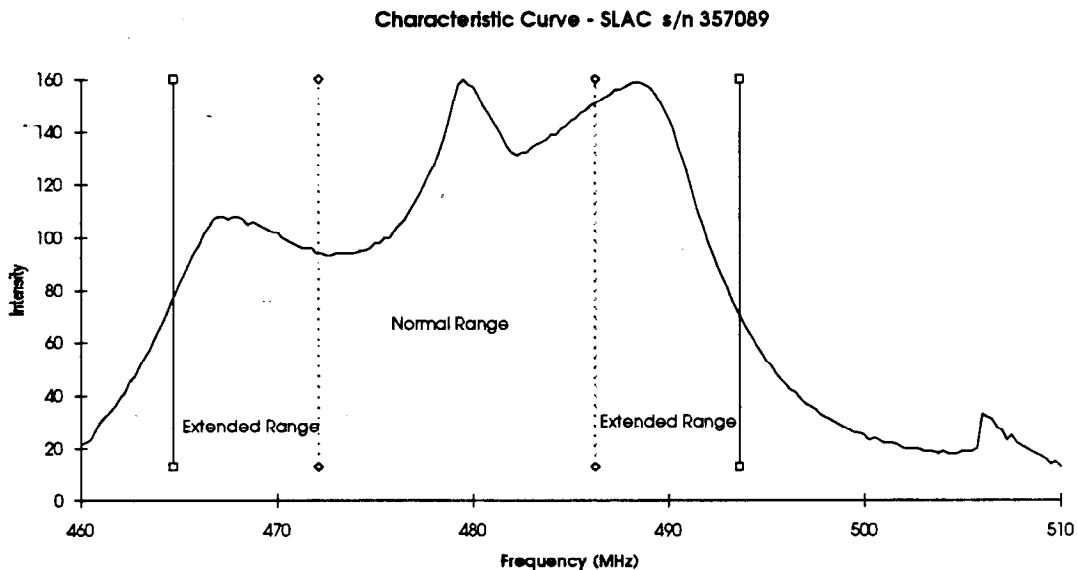


FIGURE 3. Characteristic Curve for the SLAC instrument

#### 4.2. Determining the ME5000 Windows

The Kern specification for the ME5000 Mekometer quotes a minimum distance of 20 m. This limit is chosen because every distance above 20 m can be measured unambiguously with no help from the operator. However, the Mekometer can measure shorter distances with special software and some operator assistance.

The working range of the crystal that generates the modulation frequency is 460 - 510 MHz, but for the measurement program supplied with the instrument Kern uses only the optimum portion of this bandwidth, viz 470 - 490 MHz. A distance can be measured unambiguously if two minima occur within

this optimum bandwidth, i.e., if there is an integral number of wavelengths for at least two different frequencies. In the case of most distances above 20 m there is an integral number of wavelengths (and hence a minimum) for three different frequencies. These frequencies are calculated using the basic equation for determining distance (4).

$$D = m \frac{\lambda}{2} = m \frac{c_o}{2F n}$$

(4)

where  $D$  = distance (m)  
 $m$  = number of integral wavelengths  
 $\lambda/2$  = effective wavelength ( $\approx 0.3$  m)  
 $c_o$  = velocity of light in *vacuo* (299792458 m/s)  
 $F$  = frequency (Hz)  
 $n$  = refractive index (1.000285 at 15°C, 760 mmHg)

The range of the ME5000 can be extended below 20 m using two methods. By extending the modulation bandwidth to *c.* 30 MHz two minima are found for short distances, and these distances can be determined unambiguously. For some distances below 8 m, however, only one minima exists in this frequency range. These distances can still be determined provided an approximate distance is entered (accurate to 0.1 m). Using these two methods it is possible to measure all distances greater than about 5 m, although the accuracy is inferior and the measurement time greater than for distances above 20 m.

Below about 5 m measurement is possible only within certain *windows* where there is a minimum within the extended bandwidth. The location of these windows is different for each instrument because the characteristics of each crystal determine the exact extended frequency range of the instrument. The limits of the normal and the extended bandwidth can be obtained over the ME5000 computer interface (commands SF109, SF110, SF116, SF117). With this information it is easy to calculate the locations of the measurement windows. Table 7 shows an Excel worksheet that does this, incorporating the instrument offset ADC into equation (4),

$$D = m \frac{c_o}{2F n} - ADC \quad [5]$$

The user need enter only the instrument offset, the appropriate refractive index (the table shows the case for  $n = 1.000285$ , the standard refractive index programmed into the instrument), and the bandwidth. The node  $m$  is the number of integral half-wavelengths ( $\lambda/2$ ). The distances shown under F2 and F1 are the lower and upper ends respectively of the measurement window. In the example, the extended range measurement windows overlap above 5.005 m, so all distances longer than this can be measured. Figure 4 graphically shows the extended range measurement windows.

# ME5000 Test Measurements

TABLE 7. Excel worksheet for calculating ME5000 windows

Location of Short-Range "Windows"			
SLAC s/n 357089		10-Jun-92	
Zero offset (m):	Switch setting	0.0580	
	Add. constant	0.1420	
Ref.Index	1.000285 15oC, 760mmHg		
	F1 (MHz)	F2 (MHz)	Range
Normal	473.458	487.588	14.130
Extended	466.056	494.990	28.934
Node	Extended Range		
	F2 (m)	F1 (m)	
1	0.161	0.180	
2	0.463	0.501	
3	0.766	0.823	
4	1.069	1.144	
5	1.372	1.466	
6	1.674	1.787	
7	1.977	2.109	
8	2.280	2.430	
9	2.583	2.752	
10	2.885	3.073	
11	3.188	3.395	
12	3.491	3.716	
13	3.794	4.038	
14	4.096	4.359	
15	4.399	4.681	
16	4.702	5.003	
17	5.005	5.324	
18	5.307	5.646	
19	5.610	5.967	
20	5.913	6.289	

Short-range Measurement Windows - SLAC s/n 357089

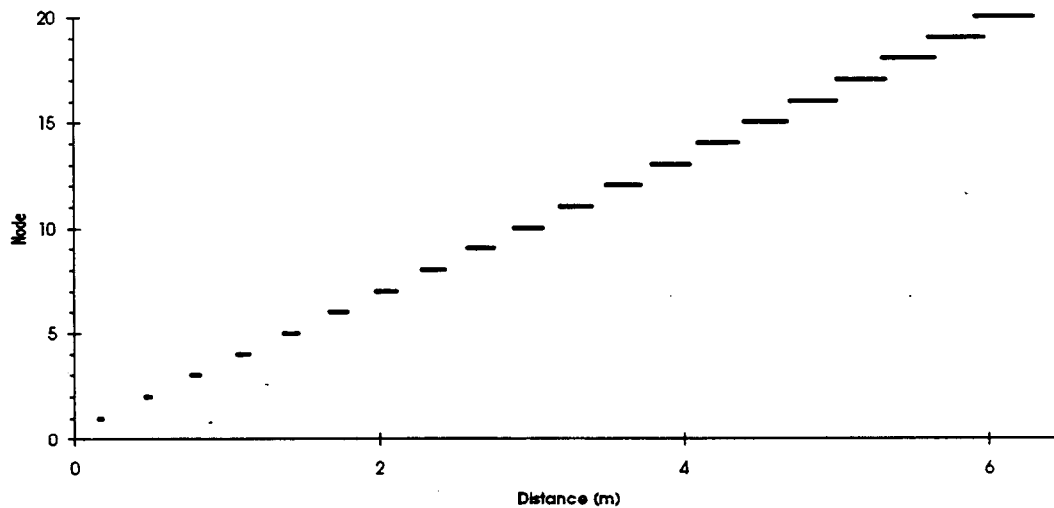


FIGURE 4. Location of short-range measurement windows

# ME5000 DATA REDUCTION

Bernard Bell, SLAC

## 1. MEKRED

The Fortran program MEKRED reduces data sets collected with the ME5000 data collection program. It is usually run under Geonet, but can be used in stand-alone mode.

## 2. THE REDUCTION PROCESS

### 2.1. Raw Distance

The raw data file (see section 3) contains both the frequencies and the raw distance calculated by the instrument from those frequencies. MEKRED calculates the distance afresh from these frequencies, using the distance only as a check.

The distance is computed in two different manners, depending on whether the distance was measured in normal mode (options "1. > 500 m" and "2. 20-500 m" in ME5000.BAS) or in short-distance mode (option "3. < 20 m").

#### 2.1.1. Normal Mode

With the exception of a few cases below 25 m where there is no middle frequency  $F_2$ , three frequencies are found for all distance above 20 m, Fig. 1.

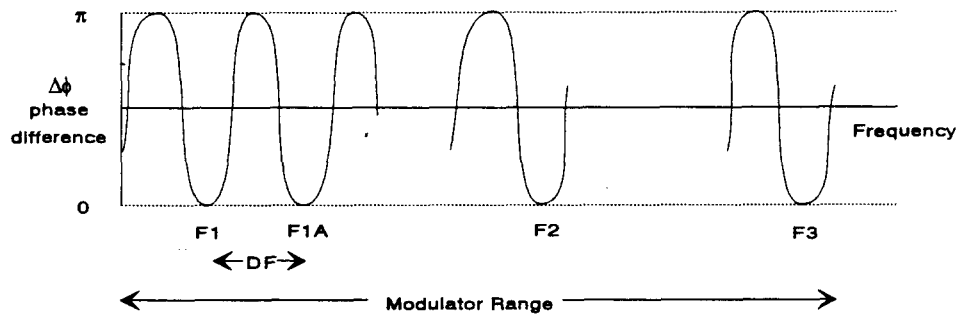


FIGURE 1. Frequencies located for distance calculation

The raw distance is calculated in the following manner, (1-4).

$$DF = \frac{F_3 - F_1}{K} \quad DF = \text{frequency difference between successive nodes} \quad (1)$$

$$N_i = \text{rnd} \left( \frac{F_i}{DF} \right) \quad K = \text{number of nodes between } F_1 \text{ and } F_3 \quad (2)$$
$$c_{stp} = \text{velocity of light in standard atmosphere} \quad (299707186.9 \text{ m/s})$$

$$D_i = \frac{N_i c_{stp}}{2F_i} - ADC \quad ADC = \text{addition constant} = 0.2 - C_i \text{ where } C_i \text{ is the constant set in the instrument} \quad (3)$$

$$D = \frac{D_1 + D_2 + D_3}{3} \quad \text{rnd} = \text{rounding function (nearest integer)} \quad (4)$$

### 2.1.2. Short Distance Mode

For all distances below 20 m only two frequencies are found. For distances below about 8 m only one frequency is found, and an approximate distance is required in order to resolve the distance.

Where two frequencies have been found the procedure is similar to that for normal range, (5-8).

$$DF = F_2 - F_1 \quad (5)$$

$$N_i = \text{rnd} \left( \frac{F_i}{DF} \right) \quad (6)$$

$$D_i = \frac{N_i c_{stp}}{2F_i} + ADC \quad (7)$$

$$D = \frac{D_1 + D_2}{2} \quad (8)$$

For the shortest distances, where only one frequency is found, an approximate distance is required, (9-11).

$$\lambda = \frac{c_{stp}}{2F_1} \quad \begin{array}{l} \lambda = \text{wavelength for frequency } F_1 \\ D_{apx} = \text{approximate distance} \end{array} \quad (9)$$

$$N_1 = \text{rnd} \left( \frac{D_{apx} + ADC}{\lambda} \right) \quad (10)$$

$$D = \lambda N_1 - ADC \quad (11)$$

## 2.2. Atmospheric Corrections

After computing the raw distance afresh from the frequencies, the first reductions applied are three different atmospheric corrections.

### 2.2.1. Refractive Index

The distance displayed on the ME5000 and recorded in the raw data file is calculated assuming a standard atmosphere, Table 1.

TABLE 1. Standard Atmosphere

Temperature	15°C
Pressure	760 mmHg, 1013.25 hPa
Moisture	none
CO <sub>2</sub> content	0.03%
Refractive Index	1.000284515

The actual meteorological parameters are measured at both ends of the line and stored in the raw data file in the following units:

Temperature: dry bulb temperature in °C.  
 Moisture: relative humidity (%) or wet bulb temperature (°C).  
 Pressure: in mmHg or mbar.

The atmospheric corrections are applied using the mean values for each of these parameters.

The ME5000 manual gives three formulae of different accuracy for calculating the refractive index. Since data reductions with MEKRED are fully automated, use is made of the most accurate of these formulae: Owen's formula. This formula requires temperature in Kelvin and the pressure subdivided into the partial pressures of dry air  $P_D$  and of water vapor  $P_W$ , both in hPa.

### 1. Partial pressure of water vapor

The partial vapor pressure of water  $e$  is calculated using either the Sprung Formula (for wet bulb temperature) (12-13), or the Goff-Gratch Formula (for relative humidity) (14-15).

#### a) Sprung Formula

$$e = E' - C (t - t') \frac{P}{755} \quad \begin{array}{ll} t & = \text{dry-bulb temperature} \\ t' & = \text{wet-bulb temperature} \end{array} \quad (12)$$

$$\text{where } E' = 10^{\left(\frac{at'}{b+t'} + c\right)} \quad \begin{array}{ll} a, b, c & = \text{constants (Table 2)} \\ E' & = \text{saturation vapor pressure (at } t') \\ e & = \text{partial pressure of the water vapor} \end{array} \quad (13)$$

$a, b, c, C$  are constants whose values differ according to whether the measurement is made "over water" (ambient temperature above freezing) or "over ice" (ambient temperature below freezing), Table 2.

TABLE 2. Constants for the Sprung Formula

	$a$	$b$	$c$	$C$
over water	7.5	237.3	0.6609	0.5
over ice	9.5	265.5	0.6609	0.43

#### b) Goff-Gratch Formula

$$\begin{aligned} \log_{10} E' &= -7.90298 \left( \frac{T_s}{T} - 1 \right) + 5.02808 \log_{10} \frac{T_s}{T} & T &= \text{dry-bulb temperature (K)} \\ &- 1.3816 \times 10^{-7} \left( 10^{\frac{11.334(1 - \frac{T}{T_s})}{T_s}} - 1 \right) & T_s &= \text{boiling point (373.16 K)} \\ &+ 8.1328 \left( 10^{-3.49149 \frac{T_s}{T} - 1} - 1 \right) + \log_{10} e_s & e_s &= 1 \text{ atmosphere (1013.25 hPa)} \\ e &= E' * RH\% \end{aligned} \quad (15)$$

### 2. Owen's Formula

$$n_t = 1 + \left\{ 80.87638002 \cdot \frac{P_D}{T} \cdot K1 + 69.09734271 \cdot \frac{P_W}{T} \cdot K2 \right\} \cdot 10^{-6} \quad (16)$$

where

$$K1 = 1 + P_D (57.9 \times 10^{-8} - \frac{9.325 \times 10^{-4}}{T} + \frac{0.25844}{T^2}) \quad (17)$$

$$K2 = 1 + P_W (1 + 3.7 \times 10^{-4} \cdot P_W) (-2.37321 \times 10^{-3} + \frac{2.23366}{T} - \frac{710.792}{T^2} + \frac{77514.1}{T^3}) \quad (18)$$

### 3. Corrected Distance

The correction for refractive index is the refractive index of the standard atmosphere  $n_s$ , divided by the refractive index of the actual atmosphere  $n_t$

$$D = D \frac{n_s}{n_t} \quad \begin{array}{l} n_s = \text{standard ref. index (1.000284515)} \\ n_t = \text{actual ref. index} \end{array} \quad (19)$$

#### 2.2.2. Beam curvature correction

The laser beam does not follow the chord between instrument and reflector, but follows a gentle arc whose radius of curvature  $\rho$  is *c.* 7-8 times larger than that of the earth  $R$ , Fig. 2.

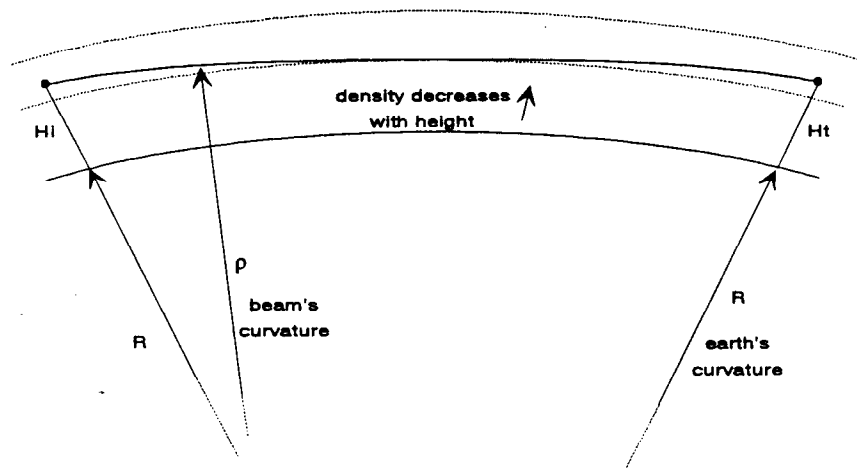


FIGURE 2. Path curvature of the ME5000 signal

The beam curvature correction  $K_1$  is the difference between the arc and the chord,<sup>1</sup>

$$K_1 = -k^2 \frac{D^3}{24 R^2} \quad \begin{array}{l} D = \text{distance} \\ R = \text{earth's radius (see sec. 3.3.1.)} \\ k = \text{coefficient of refraction} = R/\rho = 0.13 \end{array} \quad (20)$$

#### 2.2.3. Second Speed Correction

On a local scale refractive index and air density decrease more or less uniformly with increasing altitude. Following its gentle arc, the laser beam passes closer to the earth surface in its midpath, thus traversing denser air. The second speed correction  $K_2$  compensates for this higher refractive index,

$$K_2 = -k(1-k) \frac{D^3}{12 R^2} \quad (21)$$

Application of these three atmospheric corrections to the raw distance  $D_{raw}$  gives  $D_s$ , the straight-line slant distance between the instrument and the target,

<sup>1</sup> The derivation of the beam curvature correction and the second speed correction is given in most standard survey texts.

$$D_s = D_{raw} \frac{n_s}{n_i} + K_1 + K_2 \quad (22)$$

$K_1$  and  $K_2$  are very small for Mekometer distances, but more significant for distances measured with microwave EDM instruments where  $k = 0.25$  and distances are several tens of kilometers. Although these two corrections are insignificant (Table 3), they are included for the sake of completeness.

TABLE 3. Magnitude of beam curvature and second speed corrections

Distance $D$ (m)	Beam curvature $K_1$ (mm)	Second speed $K_2$ (mm)
4000	-0.001	-0.015
8000	-0.009	-0.119

### 2.3. Geometric Reductions

The next reduction applied is the geometric reduction of the slope distance to the horizontal distance at the mean height,

$$D_h = [D_s^2 - (H_i - H_t)^2]^{1/2} \quad (23)$$

$H_i$  = height of instrument  
 $H_t$  = height of target

### 2.4. Geodetic Reductions

For geodetic applications, a further stage of reductions is usually necessary in order to obtain the ellipsoidal distance on the reference figure (the curved arc distance along the reference surface at the reference height). This geodetic reduction is accomplished in two stages.

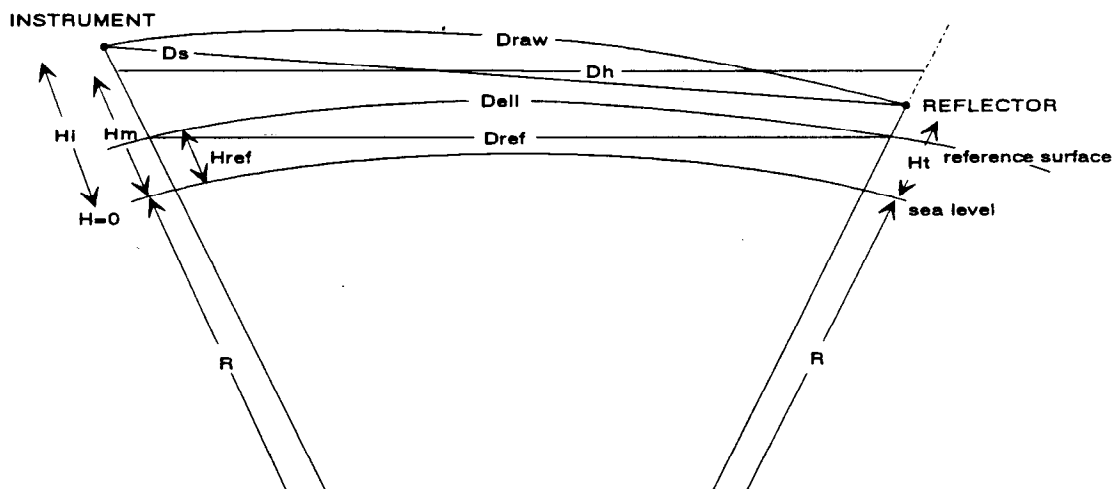


FIGURE 3. Geodetic reductions to the reference surface

#### 2.4.1. Reduction to the reference surface

As the first step, the horizontal distance at the mean height  $D_h$  is reduced to the horizontal (chord) distance  $D_{ref}$  at the reference height  $H_{ref}$

$$D_{REF} = D_h \cdot \frac{R + H_{ref}}{R + H_m} \quad (24)$$

### 2.4.2. Curvature correction

The opposite of the beam curvature correction (2.2.2), this correction reduces the distance along the chord to that along the arc,

$$D_{ELL} = D_{REF} + \frac{D^2}{24 (R + H_{ref})^2} \quad (25)$$

The magnitude of this correction is very small, Table 4. Like  $K_1$  and  $K_2$ , it is included for the sake of completeness.

TABLE 4. Chord to arc correction

Distance $D$ (m)	Correction (mm)
2000	+0.008
4000	+0.066
8000	+0.525

## 3. INPUT FILES

Since MEKRED usually runs in the Geonet environment, the management of all input and output files is performed through selection of items from menus. If the user wishes to run MEKRED outside the Geonet environment, it is only necessary to create the MEKO.PRM file and place it in the correct subdirectory.

### 3.1. Parameter File

As the user negotiates the Geonet data reduction menus, a file is prepared containing the names of all I/O files needed by Mekred. MEKRED looks for this file as C:\GEONET\PARAM\MEKO.PRM. Lines 3-7 are used by LEVRED, the level reduction program that runs immediately prior to MEKRED. Lines 1-2, 7-11 are used by MEKRED. If the user wishes to run MEKRED in stand-alone mode, it is not necessary that the files referred to in lines 3-6 even exist, provided a file of elevations exists as described in line 7.

```
N:\DATA\BLINE\MEKO\SRV9\
N:\DATA\BLINE\MEKO\SRV9\SRV9.IDX
N:\DATA\BLINE\MEKO\SRV9\INVLEVEL.IVL
N:\DATA\BLINE\MEKO\SRV9\BM.DAT
N:\CALIB\SLC\LROD\CAL1\OFFSET.IDX
C:\GEONET\TEMP\LEVRED.LST
C:\GEONET\TEMP\HEIGHTS.DAT
N:\CALIB\SLC\MEKO\CAL1\CAL1.DAT
C:\GEONET\TEMP\MEKRED.OUT
C:\GEONET\TEMP\MEKREDH.RES
C:\GEONET\TEMP\MEKREDS.RES
```

### 3.2. Raw Measurement File

The index file, identified on the second line of MEKO.PRM contains a list of all the raw measurement files for that particular survey (SRV9 in the above example). MEKRED works its way through this index file, taking one data file at a time. Two different formats have been used for these raw data files. The old format uses a fixed record length of 125 characters (125 characters per line). Since this was inconvenient for editing, the format was changed to a fixed record length of 79 characters (79 characters per line)

without the loss of any significant information. MEKFORM and MEKRED can handle both formats. The example below shows the new format.

```

06-11-92 21:53:18                                DAT
billo                                             OBS
357089                                             INS
450264      450268      0137      OVERCAST      CALM      THS
b20          0.000000                                     ST1
b19          0.000000  375624      0.14200          ST2
13.10  71.5  757.4  13.10  71.5  757.4 RH TOR      MET
473.823456 481.285278 485.016327 3  40.023600  0.00 21:55:58 21:57:49 DIS
13.10  71.5  757.4  13.10  71.5  757.4 RH TOR      MET
473.824402 481.285919 485.016174 3  40.023560  0.00 21:58:43 22:00:38 DIS
                                             FIN
b17          0.000000  375624      0.14200          ST2
13.10  71.5  757.4  13.10  71.5  757.4 RH TOR      MET
474.185913 481.665222 486.152802 8  100.037244  0.00 22:03:03 22:04:30 DIS
13.10  71.5  757.4  13.10  71.5  757.4 RH TOR      MET
474.185913 481.664886 486.152466 8  100.037290  0.00 22:05:00 22:06:40 DIS
                                             FIN

```

### 3.3. Constants File

In the Geonet environment at SLAC this file is N:\CALIB\SLC\MEKO\CAL1\CAL1.DAT. The location of this file at other labs will depend upon the configuration of Geonet.

```

REFERENCE ELLIPSOID CONSTANTS
Major-Axis  6378206.4      semi-major axis of ellipsoid
Eccentricity 0.00676866    square of first eccentricity
Latitude     37.41684906    degrees
Refheight    77.6437        normal height, meters
Ellipsoidal  2000.000       Constant added to elevations

INSTRUMENT HEIGHT CONSTANT
Instrument   Dummy          Height const
357036      0.0             0.1020  CEBAF
357037      0.0             0.1020  LBL
357046      0.0             0.1020  FNAL
357086      0.0             0.1020  ANL
357088      0.0             0.1180  BNL
357089      0.0             0.1020  SLAC

PRISM HEIGHT CONSTANTS
Prism name   Constant        Height (including adapter)
365618       0.000000        0.412000
365619       0.000000        0.412000
365630       0.000000        0.412000
374425       0.000000        0.412000
375632       0.000000        0.412000
375624       0.000000        0.412000
LBL-TH1      -0.000650        0.070000
LBL-TH2      -0.000700        0.070000
ANL-TH1      -0.001600        0.070000
ANL-TH2      -0.001500        0.070000
ANL-TH3      -0.001600        0.070000

```

This constants file contains three types of data: geodetic, instrument and reflector constants.

#### 3.3.1. Geodetic constants

The first six lines contain geodetic constants used by MEKRED. Of these, lines 2-4 are used to calculate the earth's radius. The nature of these parameters is as follows:

- a) **Major-Axis:**  $a$ , the semimajor axis of the reference ellipsoid, in meters. The reference figure used at SLAC is Clarke's ellipsoid of 1866, defined by its semimajor  $a$  and semiminor  $b$  axes,<sup>2</sup> Fig. 4.

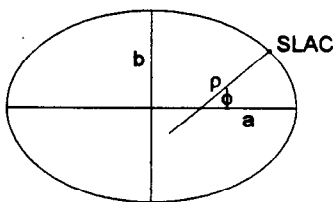


FIGURE 4. Ellipsoid

semimajor axis:  $a = 6378206.4$  m  
 semiminor axis:  $b = 6356583.8$  m

- b) **Eccentricity:**  $e^2$ , the square of the first eccentricity,

$$\text{first eccentricity: } e^2 = \frac{a^2 - b^2}{a^2} \quad (26)$$

- c) **Latitude:**  $\phi$ , the latitude of the site, in degrees.

These first three parameters are used to calculate the earth's radius of curvature at the site. Since the reference surface is a spheroid not a sphere, the radius of curvature varies with both latitude and azimuth. At a given latitude the curvature is usually expressed in the form of the two principal radii of curvature,  $\rho$  (referred to as  $M$  in some of the literature) the radius of curvature in the meridian, and  $v$  (or  $N$  in some of the literature) the radius of curvature in the prime vertical, perpendicular to the meridian (27-28). SLAC is a sufficiently small site that it is adequate to ignore the azimuth of the line, taking instead the mean radius  $R_m$  which is the geometric mean of the two principal radii, (29).

$$\rho = \frac{a(1-e^2)}{(1-e^2 \sin^2 \phi)^{3/2}} \quad (27)$$

$$v = \frac{a}{(1-e^2 \sin^2 \phi)^{1/2}} \quad (28)$$

$$R_m = \sqrt{\rho v} \quad (29)$$

The latitude given in the example file above is for the origin of the SLAC coordinate system, station 100 at the east end of the linac.

latitude:  $\phi = 37.4168^\circ$   
 mean radius:  $R_m = 6372508.16$  m

- d) **Refheight:** the reference height to which reduced ellipsoidal distances are projected. This is the normal height, in metres, above the ellipsoid.

<sup>2</sup> Ch. 8 SLC Alignment Handbook, in *SLC Design Handbook*, 1984, p. 8-31.

- e) **Ellipsoidal:** the constant added to elevations to avoid confusion between elevations and vertical cartesian coordinates (Z). The values in the elevations file read by MEKRED include this offset, which is subtracted by the program.

### 3.3.2. Instrument constants

The ninth line contains constants for the instrument. The first value is a dummy variable that is not used. The second value is the height offset. The program applies a vertical offset of 0.310m, the height of the opto-mechanical center above the base of the instrument configured with the standard LMC0500 tribrach (the true Kern tribrach). The GDF21K tribrach (the Wild version of the Kern tribrach) is an additional 0.016 m higher. The offset listed in the file is the additional offset from the base of the LMC0500 tribrach to the vertical reference point. At SLAC we use the center of the CERN socket as the vertical reference. The Kern plate on top of the standard locking pin is 0.102 m above the center of the CERN socket, Fig. 5.

MEKRED now accepts offsets for up to ten instruments.

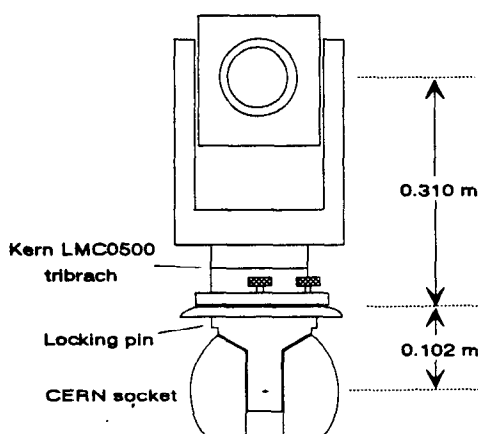


FIGURE 5. Vertical offsets for the ME5000

### 3.3.3. Reflector constants

Lines 12 onwards contain constants for the reflectors. MEKRED can handle up to thirty lines of such information. Each line has three fields. The first field is the reflector name. The second field is the horizontal addition constant. The third field is the vertical offset from the reference surface to the center of the reflector. Using SLAC's standard locking pins, the center of the regular ME5000 reflector is 0.412 m above the center of the CERN socket, Fig. 6. The center of a Taylor-Hobson ball is exactly 70 mm above the center of the CERN socket in which it sits.<sup>3</sup>

<sup>3</sup> E. Menant, The New System of SPS Reference Targets, CERN Survey Group Technical Note, 22 June 1979.

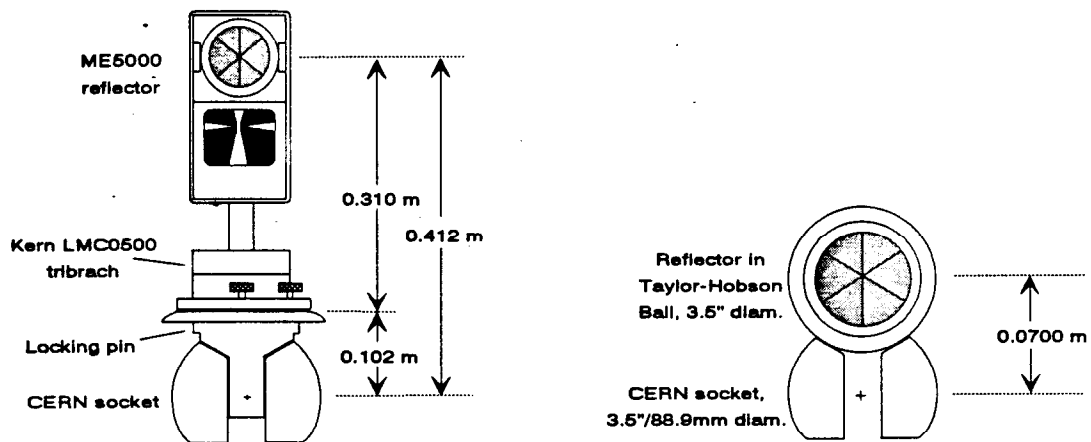


FIGURE 6. Vertical offsets for ME5000 reflectors

### 3.4. Heights File

The heights file (usually called HEIGHTS.DAT) contains a list of elevations for all instrument and target stations. These elevations include the ellipsoidal constant specified on line 6 of the constants file. This file can be up to 200 lines long.

B01	2102.77604
B02	2098.06176
B03	2095.37980
B05	2095.14142
B10	2093.97774
B12	2093.63532
B14	2093.20140
B15	2093.05902
B17	2092.67560
B19	2092.39021
B20	2092.20176

## 4. OUTPUT FILES

Three output files are created on the users hard disk in the directory C:\GEONET\TEMP.

### 4.1. MEKRED.OUT

MEKRED.OUT contains a full listing of the data reductions.

# ME5000 Data Reductions

ME5000 Mekometer Reduction File: C:\GEONET\TEMP\MEKRED.OUT  
Reduced: 07-06-92 09:07

```
=====
Station : B20
06-11-92 21:53:18
Mekometer : 357089
Add. const: .1420
Instr. Ht.: .4120
Elevation : 92.614

Observer : billo
Rman1 : miker
Rman2 : none
OVERCAST CALM
Stn Therm.: 450264
Tgt Therm.: 450268
Barometer : 0137
Ref. Ht. : 77.644
=====
```

```
Target : B19
Prism : 375624
Add.Const.: .000000
Tgt. Ht. : .41200
Elevation : 92.802

T(C) RH(%) P(mm) T(C) RH(%) P(mm) RI
13.1 71.5 757.4 13.1 71.5 757.4 284994
13.1 71.5 757.4 13.1 71.5 757.4 284994
```

```
STime ETime Dis@STP(m) Reduced Dis Corrections
21:55 21:57 40.023600 40.023043 Met : -.00002
21:58 22:00 40.023560 40.023002 Geom : -.00044
Geod : -.00009
```

Mean = 40.023022 Std Dev = .000029

```
*****
Target : B17
Prism : 375624
Add.Const.: .000000
Tgt. Ht. : .41200
Elevation : 93.088

T(C) RH(%) P(mm) T(C) RH(%) P(mm) RI
13.1 71.5 757.4 13.1 71.5 757.4 284994
13.1 71.5 757.4 13.1 71.5 757.4 284994
```

```
STime ETime Dis@STP(m) Reduced Dis Corrections
22:03 22:04 100.037244 100.035835 Met : -.00005
22:05 22:06 100.037290 100.035881 Geom : -.00112
Geod : -.00024
```

Mean = 100.035858 Std Dev = .000033

\*\*\*\*\*

The final reduced distance is the mean value for the ellipsoidal distance at the reference height. The atmospheric, geometric and geodetic corrections are listed separately for each distance.

## 4.2. MEKREDS.RES

MEKREDS.RES contains a brief listing of the reduced slope distances and their standard errors. These slope distances incorporate the three atmospheric corrections.

B20	B19	40.02356	.00003
B20	B17	100.03722	.00003
B20	B15	180.03935	.00001
B20	B14	210.03875	.00003
B20	B12	300.05123	.00022
B20	B10	370.06032	.00006
B20	B05	610.05718	.00010
B20	B03	660.06583	.00027
B20	B02	1190.08134	.00005
B20	B01	2140.10954	.00128

## 4.3. MEKREDH.RES

MEKREDH.RES contains a brief listing of the reduced geodetic distances and their standard errors. These distances incorporate all atmospheric, geometric and geodetic corrections and reductions. They are therefore the ellipsoidal distances (arc distances along the ellipsoid) at the reference height.

# *ME5000 Data Reductions*

B20	B19	40.02302	.00003
B20	B17	100.03586	.00003
B20	B15	180.03687	.00001
B20	B14	210.03586	.00003
B20	B12	300.04707	.00022
B20	B10	370.05513	.00006
B20	B05	610.04853	.00010
B20	B03	660.05646	.00027
B20	B02	1190.06357	.00005

# VARIANCE COMPONENT ANALYSIS OF BASELINE MEASUREMENTS

*Horst Friedsam, SLAC*

## 1. THE MEASUREMENT PRINCIPLE

The goal of baseline measurements is to determine the addition constant and internal accuracy for each reflector and instrument pair. To calculate the addition constant only, it is sufficient to set up three monuments in a straight line. The distance measurements from monument 1 to monuments 2 and 3, and from monument 2 to 3 are enough to obtain a unique solution for the addition constant  $c$  as shown in Figure 1. In this case  $c$  can be determined as follows:

$$D_{13} + c = D_{12} + D_{23} + 2c \quad (1-1)$$

$$c = D_{13} - (D_{12} + D_{23}) \quad (1-2)$$

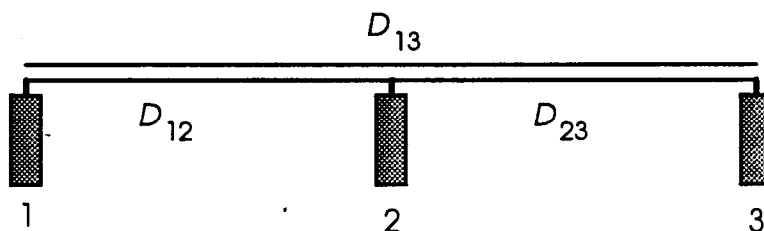


FIGURE 1. Minimum setup

To obtain an estimate of the accuracy of the instrument over a wide range of distances, more than three monuments have to be established on a baseline. The distance-dependent term of the equation describing the accuracy of the instrument, as explained in the next section, can be determined best on a long baseline. The optimum position of each monument depending on the basic measurement wavelength of the instrument can be derived by using the theory of GOLOMB rulers [1][2]. These rulers have the property that each interval appears only once on the ruler and each interval is a multiple of the smallest interval of the ruler as shown in Figure 2. This setup provides a homogeneous distribution of all possible distances for a defined measurement range without any distances of the same length.

The range of the baseline should be chosen such that the longest distance of the baseline is at least as long as the longest distance of the network to be measured. Therefore the baseline at SLAC covers distances from 40 m to 2140 m. On each monument distance measurements in all combinations are necessary. This means that each monument has to be occupied with the ME5000 and distance measurements to all other

monuments are required. This scheme, shown in Figure 3, provides a lot of redundancy for the determination of the addition constant and the variables describing the accuracy of the instrument. The method of least squares adjustment is used to estimate these values.

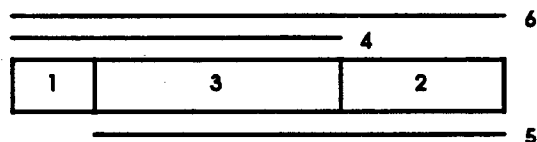


FIGURE 2. Perfect Golomb ruler

The approach used for the Mekometer calibrations does not require knowledge of the absolute distances between monuments. Absolute distances are only necessary if a scale factor between two distance measurement devices is to be determined or the scale of the instrument is in question or if the addition constant is assumed to be distance dependent.

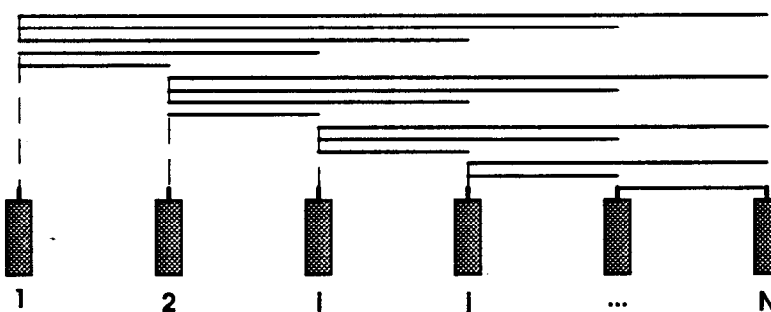


FIGURE 3. Distance measurements in all combinations

For the alignment of the SLC two distance measurement devices were used; the Kern DM503 for the measurement of the surface control network and the Distinvar for the tunnel control network. The baseline was therefore designed to allow Distinvar measurements over much of its length. A linear regression of corresponding distances for both devices is used to determine the scale factor between these instruments. The measurements with the more accurate instrument, in this case the Distinvar, are assumed to be error free. An example output of this procedure is part of the appendix (item 3). It shows the differences between DM503 and Distinvar measurements plotted versus the Distinvar distances in meters using different symbols for each station. The results of the fit are shown in the upper right corner. The coefficient  $a$  represents the y-intercept and  $b$  the slope of the regression line. The associate variances  $\sigma_a$  and  $\sigma_b$  are also displayed. The variable  $r$  represents the correlation coefficient between the DM503 and Distinvar distances and is a measure for the quality of linearity. It can vary between +1 and -1. In the case that all data points are on a straight line,  $r$  is +1. In the other extreme when no correlation between instruments exists,  $r$  is -1. The variable  $\sigma_0$  represents the variance of the residuals.

## 2. THE VARIANCE COMPONENT MODEL

### 2.1. The Mathematical model

Distance measurements on the SLAC baseline are analyzed by the program VARCOM using least squares adjustment methods. It solves for the unknown addition constant, the unknown distances from monument 1 to all other monuments, and the two variance components which describe the accuracy of the instrument.

The following observation equations can be established:

$$x_{1i} - x_{1j} + c = d_{ij} + r \quad (2-1)$$

In this equation  $c$  represents the addition constant,  $r$  the residuals,  $x_{1j}$  the unknown distance from monument 1 to all other monuments  $j$ , and  $d_{ij}$  the distance measured from monument  $i$  to  $j$  with  $i < j$ . These equations can be expressed in matrix form as follows:

$$\underline{A} \cdot \underline{x} = \underline{d} + \underline{r} \quad (2-2)$$

where  $A$  represents the coefficient matrix,  $x$  the vector of unknowns,  $d$  the vector of observations, and  $r$  the vector of residuals.

### 2.2. The Stochastic Model

A common way of creating the variance - covariance matrix is to use the formula supplied by the manufacturer of the distance measurement device which describes the accuracy of the instrument [3]. The accuracy  $m$  is usually given in the following form:

$$m = a \pm bd \quad (2-3)$$

where  $a$  represents the uncertainty of the addition constant and the cyclic error and  $b$  the uncertainty of the scale given by the frequency of the instrument for any given distance  $d$ . These values usually establish the internal accuracy of the instrument determined from repeated measurements. They do not reflect several external factors which include errors in the centering system, alignment of the reflector and measurement of the meteorological parameters. For the Kern ME5000 the accuracy range is defined by:

$$m = 0.2 \text{ mm} \pm 0.2 \text{ mm/Km} \quad (2-3a)$$

At SLAC we use an approach proposed by Koch [4]. In his paper he suggested the estimation of  $a$  and  $b$  from baseline measurements in all combinations as variance components. In this case the variance of a measured distance can be determined as follows:

$$\sigma_d^2 = \sigma_c^2 + \sigma_r^2 d^{2H} \quad (2-4)$$

Here  $\sigma_d$  represents the variance of the distance  $d$ ,  $\sigma_1$  the constant, and  $\sigma_2$  the distance-dependent term of the equation. Equations 2-3 and 2-4 can not be directly compared: 2-3 is an empirical function while 2-4 is derived through error propagation. The exponent  $H$  reflects the distance dependency of the variances. A positive exponent indicates that the variance of the measured distances increases with increasing distance. A negative exponent indicates that the variance of the measured distances decreases with increasing distance. It is usually sufficient to limit the choice of the exponent to the values +1.0, -1.0, +0.5, or -0.5.

Normally the variance - covariance matrix  $\underline{V}$  is assumed to be known with the exception of the variance  $\sigma_0$  which is estimated through the least squares process.

$$D(r) = \alpha^2 \underline{V} \quad (2-5)$$

In the case that variance components are to be estimated the stochastic model has to be expanded. Suppose the vector of residuals is composed of a linear combination of the two independent but not directly obtainable residual vectors  $r_1$  and  $r_2$ , with  $r_1$  representing the constant and  $r_2$  the distance proportional parts. Then the associated variance - covariance matrices can be described as follows:

$$D(r_1) = \sigma_1^2 \alpha_1^2 \begin{bmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \quad D(r_2) = \sigma_2^2 \alpha_2^2 \begin{bmatrix} d_{12}^{2H} & 0 & 0 & \dots \\ 0 & d_{13}^{2H} & 0 & \dots \\ 0 & 0 & d_{1i}^{2H} & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \quad (2-6)$$

where  $\sigma_1$  and  $\sigma_2$  represent the unknown variance components to be estimated and  $\alpha_1$  and  $\alpha_2$  their approximate values. The values describing the accuracy of the instrument which are usually supplied by the manufacturer can be used as approximate values (see for example equation 2-3a). The total variance - covariance  $D$  is a linear combination of both parts and is calculated as follows:

$$D = \sigma_1^2 \underline{V}_1 + \sigma_2^2 \underline{V}_2 \quad (2-7)$$

where  $\underline{V}_k$  is the product of  $\alpha_k^2$  and the associated matrix and  $\sigma_k$  are the variance components for  $k = 1, 2$ .

Using the method of least squares adjustment with additional constraints one can calculate the unknowns and their associated statistical values as follows [5]:

$$\hat{\underline{x}} = (\underline{A}^T \underline{D}^{-1} \underline{A})^{-1} \underline{A}^T \underline{D}^{-1} \underline{d} \quad (2-8)$$

$$\hat{\underline{D}}(\hat{\underline{x}}) = \sigma_o^2 (\underline{A}^T \underline{D}^{-1} \underline{A})^{-1} \quad (2-9)$$

$$\hat{\sigma}_o^2 = \frac{\underline{r}^T \underline{D}^{-1} \underline{r}}{n - u} \quad (2-10)$$

The variables with a circumflex represent the estimated values for the unknowns, the variance - covariance matrix of the unknowns, and the mean square error *a posteriori*. The degree of freedom is calculated as the difference between the number of observations  $n$  and the number of unknowns  $u$ .

The following equations are used to calculate the variance components [5]. The lengthy process of deriving these formulas is left as an exercise for the reader.

$$\hat{\underline{\sigma}} = \underline{T}^{-1} \underline{q} \quad (2-11)$$

$$\underline{T} = (tr(\underline{W} \underline{V}_i \underline{W} \underline{V}_j)) \quad \underline{q} = (d \underline{W} \underline{V}_i \underline{W} d) \quad , \quad i, j = (1, 2) \quad (2-12)$$

$$\underline{W} = \underline{D}_o^{-1} - \underline{D}_o^{-1} \underline{A} (\underline{A}^T \underline{D}_o^{-1} \underline{A})^{-1} \underline{A}^T \underline{D}_o^{-1} \quad , \quad \underline{D}_o^{-1} = \sum_{i=1}^2 \underline{V}_i \quad (2-13)$$

$$V(\hat{\sigma}_1^2) = 2t_{11} \quad \text{and} \quad V(\hat{\sigma}_2^2) = 2t_{22} \quad \text{with} \quad \underline{T}^{-1} = (t_{ij}) \quad (2-14)$$

The estimated variance components are dependent on the chosen approximate values for  $\alpha_1$  and  $\alpha_2$ . Therefore equation (2-13) shows the co-variance matrix assigned with a zero subscript. After the first iteration, the resulting variance components are used as updated approximate values for the next iteration. If this process converges then the estimated variance components will approach one [6]. At this point the products  $\sigma_1^2 \alpha_1^2$  and  $\sigma_2^2 \alpha_2^2$  will no longer change and the final result is achieved. At that point the variance of the variance components are also calculated by using equation (2-14). The estimated variance components, if they are significant, can be used to calculate the weights for a network adjustment containing distances of the calibrated instrument.

Example input and output files for the program VARCOM are given in the appendix.

The first line of the input file is used as a header for the output. If the first value on the second line is 1 then the addition constant is calculated. If this value is 0 the addition constant will not be calculated. The second value on the second line shows the maximum number of iterations followed by the value for the exponent  $H$  and a value for SMAX which is used to stop the iteration process. The process will end if  $(\sigma_1 - 1)$  and  $(\sigma_2 - 1)$  are less than SMAX. The third line starts with the number of monuments on the baseline and the approximates for  $\alpha_1$  and  $\alpha_2$ . All following lines show the distance measurements in all combinations. Each line contains the instrument and target station number and the distances reduced for geometric and meteorological effects. The program requires a numerical representation of the instrument and target stations. It also requires that the

measurements be input in a specific order. The instrument station which is used to measure to all other target stations has to be the first data set. For each instrument station the distance measurements have to be input in ascending order.

The output file shows the date and time when the data was processed, which input file was used, the header line of the input file and indicates if the calculations were done with or without the addition constant. It also shows the number of baseline monuments, the value for the exponent  $H$  used and the start values for  $\alpha_1$  and  $\alpha_2$ . The next section contains the results after  $n$  iterations starting with the mean square error *a posteriori*, the variance components  $\sigma_1^2$ ,  $\sigma_2^2$  and their estimated variances, the adjusted distances with their r.m.s., and the addition constant if calculated. The variable  $S_0$  represents the approximate distance and  $dS$  the addition to obtain the adjusted distance. Finally, the last section contains each measured distance with its residual and the estimated mean square error  $M(s)$  according to equation (2-4). The weights  $1/M(s)^2$  are also calculated.

### 2.3. Extensions to the mathematical and stochastic model

The previous section describes all the unknowns and variance components which are calculated by the program VARCOM used at SLAC for the analysis of baseline measurements. In a more recent paper Koch [7] expanded the mathematical and stochastic model to include the case of distance dependent addition constants and cyclic errors which can be determined if absolute distances are known. It also deals with the situation in which an instrument with superior accuracy is used as reference for an instrument with inferior accuracy. The mathematical model (2-1) can be expanded as follows:

$$x_{1i} - x_{1j} + C + K = d_{ij} + r \quad \text{with} \quad (2-15)$$

$$C = c_0 + c_1 d_{ij} + c_2 d_{ij}^2 + c_3 d_{ij}^3 \quad \text{and} \quad (2-16)$$

$$K = k_{11} \cos(\beta) + k_{12} \cos(2\beta) + k_{21} \sin(\beta) + k_{22} \sin(2\beta) \quad \text{and} \quad (2-17)$$

$$\beta = 2\pi \frac{\text{mod}(d_{ij}, F)}{F} \quad (2-18)$$

Equation (2-16) represents in polynomial form the distance-dependent addition constant  $C$  of the instrument to be calibrated. Equation (2-17) shows the first terms of a Fourier polynomial to describe the cyclic phase error  $K$ . The unknowns  $c_0$  through  $c_3$  and  $k_{11}$  through  $k_{22}$  of the instrument to be calibrated are summarized in the vector of unknowns  $x_c$ , while the unknowns of the superior instrument are summarized in the vector  $x_s$ . Likewise the matrix of coefficients is split into a section containing the coefficients of the instrument to be calibrated and a section containing the coefficients of the instrument which is used as reference. Therefore the following observation equations in matrix form can be derived.

$$\begin{pmatrix} \underline{A_s} \\ \underline{A_c} \end{pmatrix} \begin{pmatrix} \underline{x_s} \\ \underline{x_c} \end{pmatrix} = \begin{pmatrix} \underline{d_s} \\ \underline{d_c} \end{pmatrix} + \begin{pmatrix} \underline{r_s} \\ \underline{r_c} \end{pmatrix} \quad (2-19)$$

The subscript  $s$  indicates measurements with the superior instrument and the subscript  $c$  is used to reference the measurements with the instrument to be calibrated. Both instruments are subject to having a constant and distance-dependent part which are estimated as variance components. In analogy to equation (2-6) the variance - covariance matrix can be extended as follows:

$$D = \sigma_1^2 \alpha_1^2 \begin{bmatrix} \underline{I_s} & \underline{O} \\ \underline{O} & \underline{O} \end{bmatrix} + \sigma_2^2 \alpha_2^2 \begin{bmatrix} \underline{O} & \underline{O} \\ \underline{O} & \underline{I_c} \end{bmatrix} + \sigma_3^2 \alpha_3^2 \begin{bmatrix} \underline{D_s} & \underline{O} \\ \underline{O} & \underline{O} \end{bmatrix} + \sigma_4^2 \alpha_4^2 \begin{bmatrix} \underline{O} & \underline{O} \\ \underline{O} & \underline{D_c} \end{bmatrix} \quad (2-20)$$

$$\text{or } D = \sigma_1^2 V_1 + \sigma_2^2 V_2 + \sigma_3^2 V_3 + \sigma_4^2 V_4 \quad (2-21)$$

The matrices  $D_c$  and  $D_s$  are created in the same way as the second part of equation (2-6) using different exponents for both instruments for the adjustment of the distance dependent variances. The variance components  $\sigma_1$  and  $\sigma_2$  represent respectively the constant terms of the superior instrument and of the instrument to be calibrated. Likewise,  $\sigma_3$  and  $\sigma_4$  are the distance dependent terms for both instruments. Depending on the type of baseline measurements this extended model has more flexibility for the analysis of the measurements. Very seldom is it necessary to determine all unknowns of the model. A significance test can be used to determine those unknowns to be included in the adjustment.

If  $\sigma_3$  and  $\sigma_4$  are set to zero then only the variance components for the constant terms of both instruments will be calculated. In the case that  $\sigma_1$  and  $\sigma_2$  are set to one, only the variance components of the instrument to be calibrated will be determined while the variances for the superior instrument do not change during the iteration process. If the defined variances  $V_1 + V_3$  are much smaller than the estimated variances  $\sigma_2^2 V_2 + \sigma_4^2 V_4$  then the distances of the superior instrument are considered to be quasi error free and thus define the absolute distances of the baseline. In the opposite case when the defined variances are large in comparison to the estimated variances then both instruments are evaluated with unknown absolute distances of the baseline.

### 3. CONCLUSION

The currently implemented mathematical model for the analysis of Mekometer distances as explained in section two is sufficient but should be extended to the model described in section 2.3. if other EDM instruments with less accuracy have to be calibrated and compared to Mekometer data. The expanded model covers most possible cases for analyzing baseline measurements and is therefore more flexible

All ME5000 instruments participating in the 1992 measurement campaign on the SLAC baseline fell within the defined accuracy range provided by KERN.

### REFERENCES

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- [2] Bloom, G. S. and Golomb, S. W. (1978) "Numbered complete graphs, Unusual rulers and assorted applications, in theory and applications of graphs." (ed. Y. Alavi and D. R. Lick) Springer Verlag Lecture Notes in Mathematics 642, p 53-65.
- [3] Fröhlich, H. (1982) "Varianzkomponentenschätzung für elektrooptische Distanzmessungen auf Eichlinien." AVN, 89, p 411-417.
- [4] Koch, K. R. (1978), "Schätzung von Varianzkomponenten.", AVN, 85, p 264-269.
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- [7] Koch, K. R. (1987), "Zur Auswertung von Streckenmessungen auf Eichlinien mittels Varianzkomponentenschätzung.", AVN, 2, p 63-71.

## APPENDIX

## 1. Example input file for VARCOM

```
TEST EXAMPLE FRÖHLICH MEKOMETER MEAS. (D2) MUNICH
1 20 1.0 0.00001
7 1.00 0.01
1 2 26.50860
1 3 161.51630
1 4 243.01060
1 5 431.98070
1 6 485.52540
1 7 540.01610
2 3 135.00800
2 4 216.50260
2 5 405.47220
2 6 459.01660
2 7 513.50830
3 4 81.49540
3 5 270.46480
3 6 324.00970
3 7 378.50070
4 5 188.97030
4 6 242.51510
4 7 297.00590
5 6 53.54590
5 7 108.03670
6 7 54.49150
```

## Variance Component Analysis

### 2. Example output file for VARCOM

DATE : 07-14-89  
TIME : 14:56

INPUT-FILENAME : VARCOM  
FILETYPE : INPUT

PROGRAM VARCOM

TEST EXAMPLE FRÖHLICH MEKOMETER MEAS. (D2) MUNICH

VARIANCE COMPONENT ESTIMATION

=====

COMPUTATION WITH ADDITION CONSTANT

-----

NUMBER OF BASELINE POINTS = 7

ALPHA1 = 1.00000 ALPHA2 = 0.01000 H = 1.00

$M(s)^{**2} = SIGMA1^{**2} + SIGMA2^{**2} * S^{**2} * H$

\*\*\*\*\*

RESULTS AFTER 11 ITERATIONS

M0 = 1.000

SIGMA1\*\*2 = 0.023 (mm) M(SIG1\*\*2) = 0.022 (mm)  
SIGMA2\*\*2 = 0.310 (mm/Km) M(SIG2\*\*2) = 0.313 (mm/Km)

FROM	TO	S0 (m)	dS (mm)	S0 + dS	R.M.S. (mm)
1	2	26.50860	-0.517	26.50808	0.115
1	3	161.51630	-0.850	161.51545	0.129
1	4	243.01060	-0.543	243.01006	0.150
1	5	431.98070	-1.171	431.97953	0.169
1	6	485.52540	-0.839	485.52456	0.197
1	7	540.01610	-0.673	540.01543	0.233

ADDITION CONSTANT C = -0.702 (mm) M(c) = 0.087

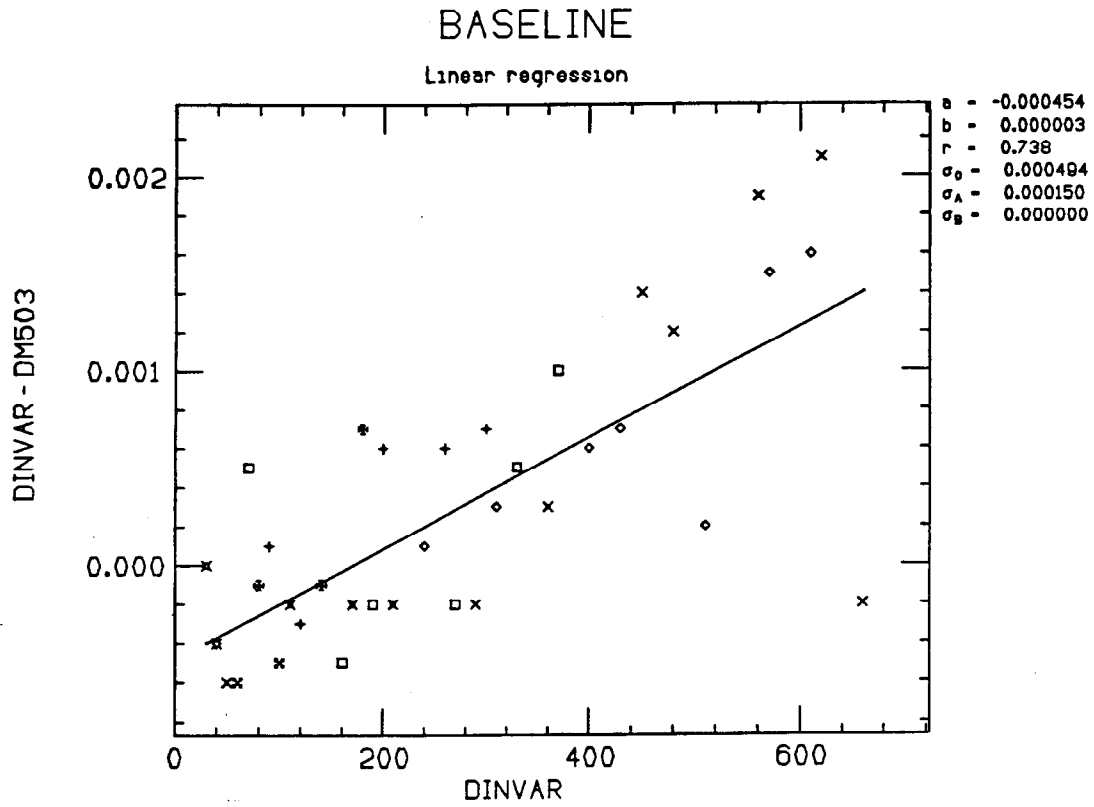
*... continued on next page ...*

## VARCOM output file [cont.]

FROM	TO	DIST(m)	RESIDUALS (mm)	M(s) (mm)	1 / (M(s) **2)
1	2	26.50860	0.185	0.154	42.338
1	3	161.51630	-0.148	0.177	31.765
1	4	243.01060	0.159	0.204	23.986
1	5	431.98070	-0.469	0.285	12.316
1	6	485.52540	-0.137	0.311	10.372
1	7	540.01610	0.029	0.337	8.794
2	3	135.00800	0.069	0.170	34.427
2	4	216.50260	0.076	0.195	26.372
2	5	405.47220	-0.052	0.273	13.455
2	6	459.01660	0.590	0.298	11.280
2	7	513.50830	-0.254	0.324	9.518
3	4	81.49540	-0.091	0.160	39.280
3	5	270.46480	-0.019	0.215	21.712
3	6	324.00970	0.112	0.235	17.884
3	7	378.50070	-0.021	0.260	14.756
4	5	188.97030	-0.126	0.186	29.018
4	6	242.51510	0.105	0.204	24.029
4	7	297.00590	0.172	0.225	19.715
5	6	53.54590	-0.166	0.156	41.170
5	7	108.03670	-0.100	0.164	37.014
6	7	54.49150	0.068	0.156	41.116

## Variance Component Analysis

### 3. Example output for a linear regression fit between Distinvar and DM503 measurements



# ME5000 RESULTS

Bernard Bell, SLAC

## 1. FREQUENCY MEASUREMENTS

### 1.1. The Characteristic Curve

The *characteristic curve*, the bandwidth profile of the modulation cavity output, has been measured for five instruments, Fig. 1. The curves of the two SSC instruments have a similar shape to the nominal curve shown in the Kern literature, whereas the other three curves show a shoulder at the lower end of the frequency range. It is this shoulder that gives these instruments a wider extended bandwidth, Table 3. The peak intensity of the five curves varies considerably, Table 1.

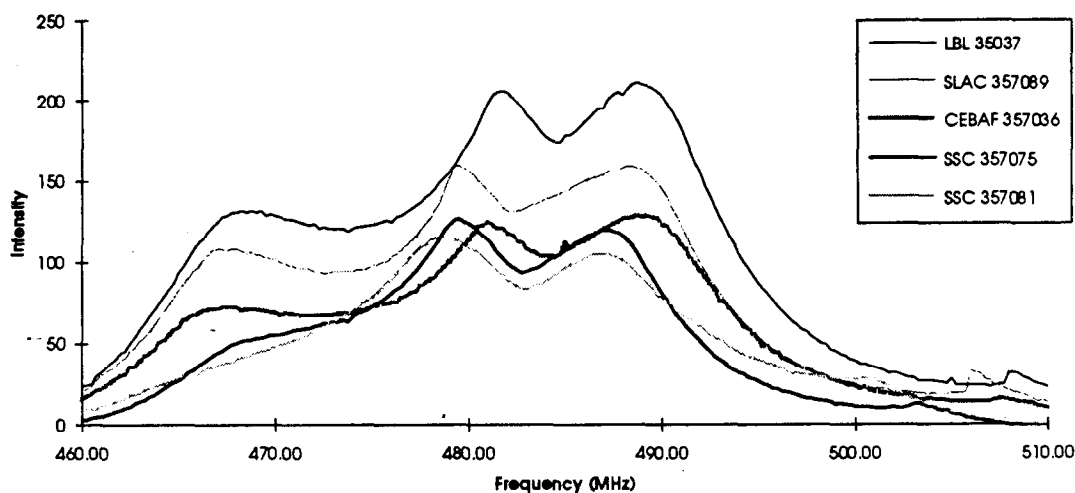


FIGURE 1. The Characteristic Curve for five instruments

TABLE 1. Peak intensities on the characteristic curves

Instrument		Min.	Max.
LBL	357037	23	211
SLAC	357089	12	156
CEBAF	357036	10	129
SSC	357075	0	123
SSC	357081	0	112

As the instrument warms up, the characteristic curve exhibits a slight change in shape, Fig. 2. The result of this change in the modulator output is a downward shift in the lower and upper limits of the modulator bandwidth, Table 2. All instruments show a similar downward shift of about 2 MHz in the first two hours after the instrument is switched on. This has no effect on measurement accuracy but does change the location of the short-range measurement windows.

## ME5000 Results

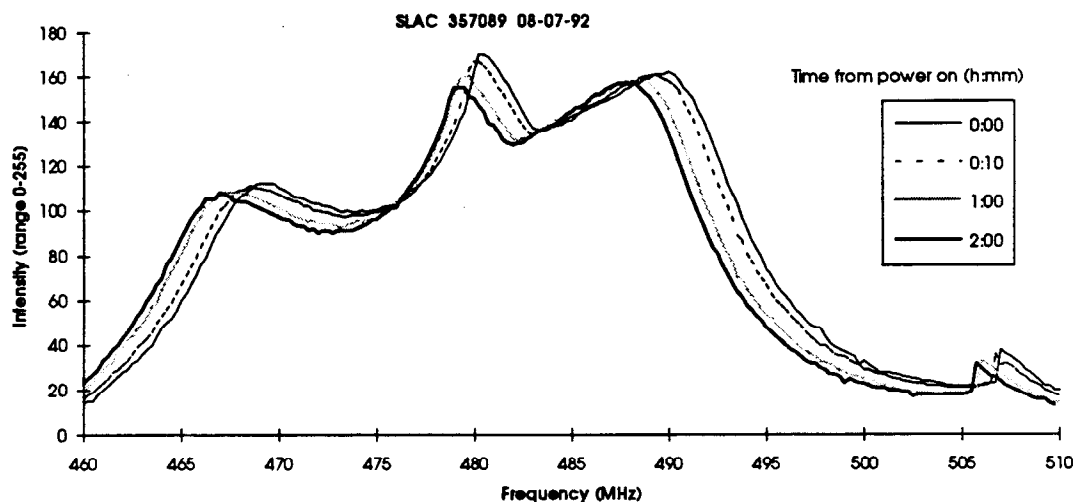


FIGURE 2. Change of characteristic curve with time

TABLE 2. Change of modulator bandwidth with time: instrument 357089 (SLAC)

Time h:mm	Normal Range (MHz)			Extended Range (MHz)		
	Lower	Upper	Range	Lower	Upper	Range
0:00	473.458	487.588	14.130	466.056	494.990	28.934
0:20	472.785	486.915	14.130	465.383	494.317	28.934
1:00	472.112	486.242	14.130	464.710	493.644	28.934
2:00	471.439	485.569	14.130	464.037	492.971	28.934

## 1.2. Modulator Bandwidth

The nominal bandwidths are 15 MHz for normal and 30 MHz for extended modes. Measurements show that the normal bandwidth is always either 14.130 or 14.803 (the difference between these two values is the spacing between measurements during the modulator calibration in which the bandwidth is determined). The extended bandwidth, by contrast, varies considerably between instruments, Table 3.

TABLE 3. Modulator Bandwidths

Lab	Instrument s/n	Date	Normal Range (MHz)			Extended Range (MHz)		
			Lower	Upper	Range	Lower	Upper	Range
LBL	357037	01-06-92	472.112	486.915	14.803	465.383	494.644	28.261
Fermilab	357046	01-13-92	476.149	490.952	14.803	474.130	492.971	18.841
CEBAF	357036	01-27-92	476.822	490.952	14.130	473.458	494.317	20.859
Argonne	357086	02-18-92	474.130	488.934	14.804	468.747	494.317	25.570
Brookhaven	357088	03-16-92	476.149	490.280	14.131	472.785	493.644	20.859
SLAC	357089	06-10-92	472.112	486.242	14.130	464.710	493.644	28.934
SSC	357075	08-03-92	473.458	487.588	14.130	470.766	490.280	19.513
SSC	357081	08-06-92	472.785	486.915	14.130	469.420	490.280	20.859

## 1.3. Scale Factor

The frequency of each instrument is measured by Kern (Leica) in Switzerland prior to shipping, and the result is shipped with the instrument in the form of a calibration certificate. This calibration consists of a single measurement at c. 480 MHz. At SLAC a more comprehensive set of frequency measurements

has been made. The reference standard is a Hewlett-Packard HP 5342A microwave frequency counter. The frequency output of each ME5000 was compared with the counter at 2 MHz intervals in the range 466-492 MHz, giving 14 values across the entire modulation range of the instrument. The final result was the mean of these 14 values. Table 4 shows these results together with the single value supplied by Leica, where available. A positive value indicates that the frequency set on the ME5000 is higher than the frequency registered by the counter. The agreement between the SLAC measurements and the Leica measurements is close.

Table 4. Comparison against frequency counter

Lab	s/n	Date	Diff. Hz	Scale ppm	Leica data	
					Date	Hz
LBL	357037	01-03-92	+232	+0.5		N/A
Fermilab	357046	01-13-92	+101	+0.2		N/A
CEBAF	357036	01-27-92	+174	+0.4	8-18-88	+20
Argonne	357086	02-18-92	+24	+0.05		N/A
Brookhaven	357088	03-16-92	+166	+0.3	3-04-92	+180
SLAC	357089	06-10-92	+18	+0.04	3-12-92	+40
SSC	357075	08-11-92	+22	+0.04	3-12-92	-30
SSC	357081	08-07-92	+62	+0.1	5-25-92	+70

The crystal of the CEBAF instrument (357036) exhibits signs of aging, showing a steadily increasing positive discrepancy in frequency when compared against a frequency standard. This instrument has been the most heavily used in the nearly four years since its acquisition.

## 2. BASELINE MEASUREMENTS

A total of 110 distances were measured with each instrument: all 55 distances between the eleven pillars measured in both directions. The set of measurements from B20 to B01 is called the *forward* set (it is measured from east to west starting at the end closest to the center of the SLAC site). The set measured eastwards from B01 to B20 is the *back* or *reverse* set. Each distance is the mean of two individual measurements.

### 2.1. Comparison of Forward and Back Measurements

The first analysis after the MEKRED data reductions was to compare the forward and reverse measurements for repeatability, Table 5 and Fig. 3.

ME5000 Results

TABLE 5. Comparison of Forward and Back measurements

-From	To	Distance	LBL	FNAL	CEBAF	ANL	BNL	SLAC	SSC	SSC
			357037	357046	357036	357086	357088	357089	357081	357075
B20	B19	40	-0.35	0.03	-0.05	0.11	-0.43	-0.12	-0.05	-0.14
	B17	100	-0.30	0.04	-0.10	-0.02	-0.03	0.19	0.26	-0.09
	B15	180	0.43	-0.08	0.10	0.13	0.08	0.18	0.14	0.12
	B14	210	-0.10	0.41	0.28	0.05	0.01	0.05	0.47	-0.19
	B12	300	-0.56	0.17	0.08	-0.01	0.02	0.30	-0.23	-0.04
	B10	370	0.03	0.44	-0.07	-0.25	-0.39	0.02	-0.08	-0.63
	B05	610	1.09	0.73	-0.12	-0.75	0.01	-0.44	0.08	-0.08
	B03	660	0.65	0.53	0.41	0.63	0.15	-0.25	0.11	-0.06
	B02	1190	1.37	0.90	-0.41	0.38	1.11		0.38	-0.46
	B01	2140	0.61	2.09	0.29	0.92	1.87		0.09	0.22
B19	B17	60	-0.11	0.05	-0.05	-0.06	-0.17	-0.03	-0.06	-0.18
	B15	140	0.13	0.01	0.08	-0.08	-0.16	0.07	-0.20	0.04
	B14	170	-0.02	0.14	0.19	-0.18	-0.20	-0.06	-0.14	-0.47
	B12	260	-0.03	0.11	0.09	-0.01	-0.09	-0.04	-0.12	0.42
	B10	330	-0.07	0.11	-0.20	-0.11	-0.08	-0.64	-0.46	-0.33
	B05	570	0.62	0.22	-0.20	-0.75	0.01	0.51	-0.74	-0.72
	B03	620	0.02	0.16	-0.15	0.78	0.01	1.26	0.03	-0.14
	B02	1150	0.32	0.66	0.10	0.86	0.86		-0.19	-1.96
	B01	2100	0.50	0.62	0.24	3.08	0.96		0.07	1.39
B17	B15	80	0.11	0.08	0.07	0.16	-0.34	0.10	0.93	0.15
	B14	110	-0.15	0.14	0.01	-0.23	-0.25	-0.12	-0.01	-0.05
	B12	200	0.03	0.03	0.09	0.26	-0.08	-0.25	0.22	0.41
	B10	270	-0.16	0.18	-0.27	0.11	-0.20	0.06	0.18	-0.32
	B05	510	0.21	0.25	-0.14	1.10	-0.45	0.04	0.18	-0.10
	B03	560	0.12	-0.23	0.18	1.02	-0.50	0.45	0.59	1.17
	B02	1090	-0.62	0.04	0.04	1.54	-0.63		-0.25	0.47
	B01	2040	-0.38	1.03	0.13	4.00	1.07		2.97	1.09
B15	B14	30	0.05	-0.07	0.07	0.10	-0.29	-0.18	0.05	-0.12
	B12	120	0.05	-0.08	0.12	0.13	-0.16	0.00	0.05	-0.21
	B10	190	-0.05	-0.11	-0.32	0.04	-0.42	-0.03	0.17	-0.58
	B05	430	0.44	0.15	-0.06	0.28	-0.93	-0.41	1.10	-0.69
	B03	480	0.07	-0.11	-0.04	0.33	-1.12	-0.20	0.52	-0.37
	B02	1010	-0.69	0.03	0.02	0.33	-0.93		0.14	-0.37
	B01	1960	-1.28	-0.84	-1.63	1.70	-2.79		2.52	0.61
B14	B12	90	-0.02	-0.14	0.06	-0.06	-0.34	0.03	-0.11	-0.03
	B10	160	-0.10	0.00	-0.22	0.00	-0.18	0.09	-0.01	-0.18
	B05	400	0.45	0.01	-0.28	-0.29	-0.51	-0.09	-0.36	0.43
	B03	450	-0.03	-0.36	-0.07	0.35	-0.72	-0.07	-0.53	0.05
	B02	980	0.22	-0.34	0.71	0.13	-0.90		0.20	0.20
	B01	1930	0.41	-1.70	0.47	1.22	-0.43		-2.28	1.36
B12	B10	70	-0.05	-0.02	-0.15	0.36	-0.17	-0.07	-0.39	-0.33
	B05	310	0.28	0.19	-0.12	0.23	-0.06	0.01	-0.26	-0.77
	B03	360	-0.16	0.26	0.00	0.41	0.02	0.10	-0.16	0.19
	B02	890	-0.30	-0.10	0.50	-0.07	-0.15		-0.73	-2.29
	B01	1840	-0.58	1.47	-0.19	0.99	-0.59		0.76	0.45
B10	B05	240	0.55	-0.20	-0.05	0.23	-0.48	-0.12	0.08	-0.55
	B03	290	0.19	-0.22	0.04	0.33	-0.30	0.14	0.12	-0.10
	B02	820	0.21	0.18	0.50	0.21	0.17		-0.04	-0.30
	B01	1770	0.73	0.37	-0.02	2.09	1.59		2.42	1.00
B05	B03	50	-0.18	0.12	0.06	0.10	-0.43	-0.08	-0.18	0.07
	B02	580	-0.65	0.07	0.19	0.42	-0.10		0.34	0.00
	B01	1530	-2.88	0.10	0.14	1.75	0.19		0.50	1.54
B03	B02	530	-0.23	0.47	-0.06	-0.04	-0.34		-0.19	0.29
	B01	1480	-1.63	-1.06	0.28	0.62	0.37		0.36	0.89
B02	B01	950	0.56	-2.35	-0.09	-0.08	0.97		0.95	0.00
Max.			1.37	2.09	0.71	4.00	1.87	1.26	2.97	1.54
Min.			-2.88	-2.35	-1.63	-0.75	-2.79	-0.64	-2.28	-2.29
Std. err.			0.64	0.63	0.31	0.84	0.69	0.31	0.77	0.68

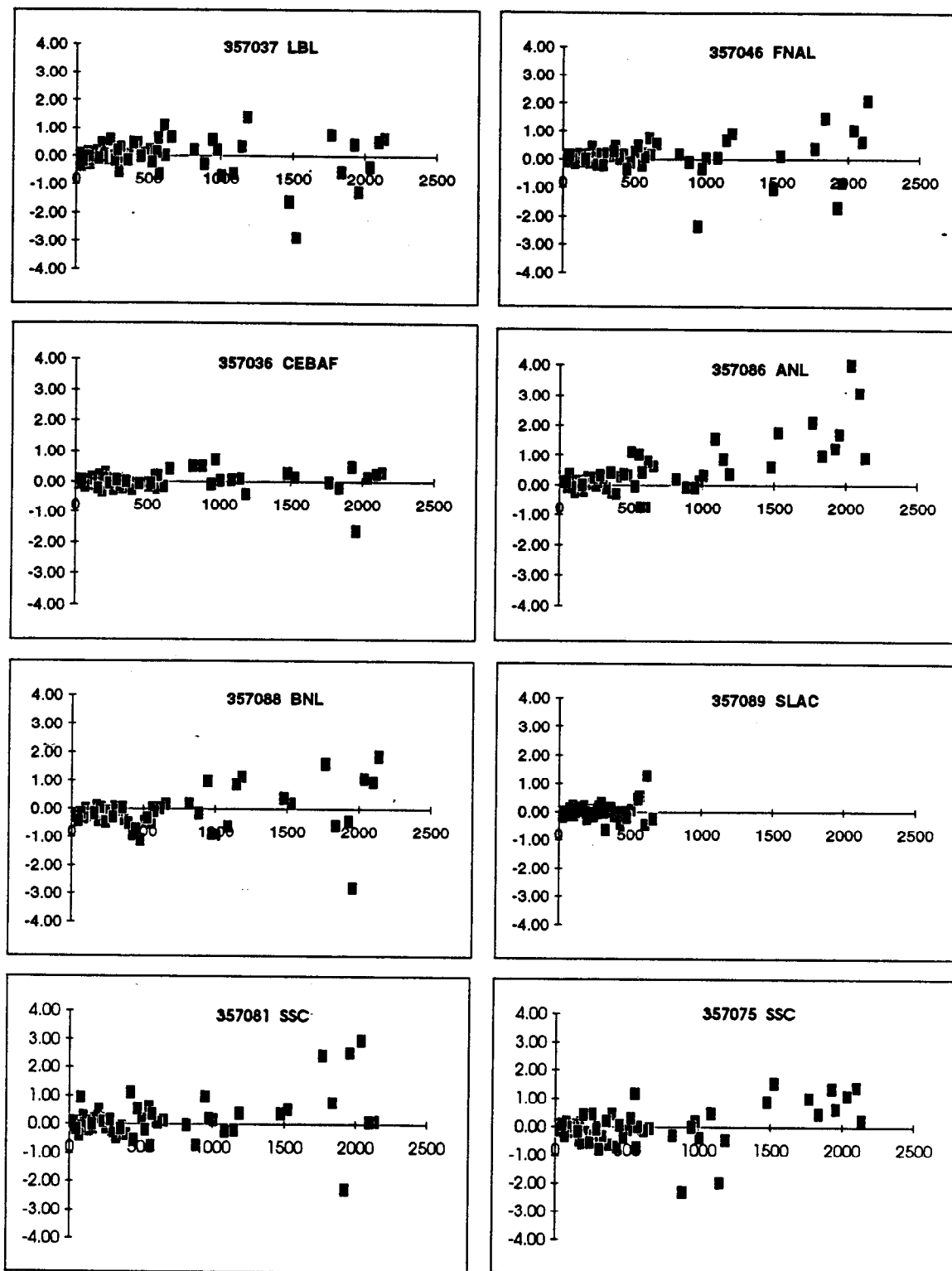


FIGURE 3. Comparison of Forward and Back measurements

## 2.2. Comparison with the average

The next evaluation was to compare each set of measurements against the average of all eight sets, Tables 6, 7.

TABLE 6. Baseline results of eight instruments, and the average

From	To	LBL	FNAL	CEBAF	ANL	BNL	SLAC	SSC	SSC	Average*
		357037	357046	357036	357086	357088	357089	357081	357075	
		1-92	1-92	1-92	2-92	3-92	6-92	8-92	8-92	
B20	B19	40.02259	40.02260	40.02256	40.02225	40.02250	40.02302	40.02339	40.02361	40.02289
	B17	100.03598	100.03592	100.03584	100.03580	100.03563	100.03586	100.03630	100.03671	100.03602
	B15	180.03660	180.03694	180.03662	180.03653	180.03670	180.03687	180.03665	180.03687	180.03671
	B14	210.03538	210.03520	210.03504	210.03544	210.03571	210.03586	210.03625	210.03622	210.03575
	B12	300.04654	300.04596	300.04617	300.04643	300.04659	300.04707	300.04740	300.04726	300.04682
	B10	370.05463	370.05442	370.05450	370.05483	370.05471	370.05513	370.05496	370.05546	370.05493
	B05	610.04740	610.04765	610.04799	610.04898	610.04827	610.04853	610.04875	610.04905	610.04860
	B03	660.05306	660.05403	660.05400	660.05518	660.05545	660.05646	660.05652	660.05712	660.05579
	B02	1190.06121	1190.06226	1190.06236	1190.06244	1190.06288	1190.06357	1190.06337	1190.06440	1190.06317
	B01	2140.07479	2140.07422	2140.07617	2140.07636	2140.07528	2140.07662	2140.07926	2140.07904	2140.07712
B19	B20	40.02224	40.02263	40.02251	40.02236	40.02207	40.02290	40.02334	40.02347	40.02278
	B17	60.01331	60.01341	60.01330	60.01344	60.01318	60.01303	60.01291	60.01328	60.01319
	B15	140.01421	140.01427	140.01415	140.01436	140.01449	140.01396	140.01332	140.01330	140.01393
	B14	170.01270	170.01279	170.01258	170.01320	170.01341	170.01301	170.01242	170.01277	170.01290
	B12	260.02371	260.02362	260.02358	260.02394	260.02448	260.02403	260.02370	260.02372	260.02391
	B10	330.03196	330.03203	330.03194	330.03216	330.03228	330.03224	330.03172	330.03160	330.03199
	B05	570.02543	570.02562	570.02555	570.02644	570.02597	570.02516	570.02503	570.02538	570.02559
	B03	620.03130	620.03174	620.03169	620.03262	620.03327	620.03200	620.03254	620.03285	620.03250
	B02	1150.03940	1150.03990	1150.03979	1150.03998	1150.03985	1150.04012	1150.03995	1150.04174	1150.04024
	B01	2100.05186	2100.05288	2100.05291	2100.05188	2100.05461	2100.05434	2100.05551	2100.05343	2100.05378
B17	B20	100.03568	100.03596	100.03574	100.03578	100.03560	100.03605	100.03656	100.03662	100.03606
	B19	60.01320	60.01346	60.01325	60.01338	60.01301	60.01300	60.01285	60.01310	60.01301
	B15	80.00071	80.00094	80.00083	80.00078	80.00130	80.00103	79.99995	80.00007	80.00066
	B14	109.99928	109.99957	109.99935	109.99998	110.00019	110.00010	109.99917	109.99933	109.99969
	B12	200.01031	200.01052	200.01036	200.01042	200.01126	200.01120	200.01034	200.01033	200.01065
	B10	270.01868	270.01873	270.01872	270.01858	270.01909	270.01909	270.01838	270.01859	270.01874
	B05	510.01218	510.01243	510.01212	510.01230	510.01293	510.01241	510.01176	510.01196	510.01225
	B03	560.01812	560.01866	560.01836	560.01890	560.02025	560.01987	560.01947	560.01924	560.01935
	B02	1090.02712	1090.02650	1090.02647	1090.02563	1090.02820	1090.02685	1090.02711	1090.02669	1090.02683
	B01	2040.03925	2040.03845	2040.03867	2040.03758	2040.04028	2040.04112	2040.04184	2040.04170	2040.04020
B15	B20	180.03703	180.03686	180.03672	180.03666	180.03678	180.03705	180.03679	180.03699	180.03683
	B19	140.01434	140.01428	140.01423	140.01428	140.01433	140.01403	140.01312	140.01334	140.01389
	B17	80.00082	80.00102	80.00090	80.00094	80.00096	80.00113	80.00088	80.00022	80.00084
	B15	29.99838	29.99882	29.99847	29.99875	29.99902	29.99910	29.99903	29.99932	29.99895
	B12	120.00938	120.00980	120.00953	120.00974	120.01024	120.01023	120.01024	120.01060	120.01010
	B10	190.01779	190.01814	190.01803	190.01792	190.01822	190.01820	190.01816	190.01884	190.01823
	B05	430.01134	430.01174	430.01168	430.01212	430.01251	430.01161	430.01143	430.01260	430.01199
	B03	480.01743	480.01774	480.01782	480.01863	480.01983	480.01904	480.01919	480.02025	480.01913
	B02	1010.02641	1010.02616	1010.02623	1010.02596	1010.02754	1010.02709	1010.02659	1010.02793	1010.02689
	B01	1960.04001	1960.04056	1960.04094	1960.03914	1960.04218	1960.04045	1960.03968	1960.04206	1960.04074
B14	B20	210.03528	210.03561	210.03532	210.03549	210.03572	210.03591	210.03672	210.03603	210.03587
	B19	170.01268	170.01293	170.01277	170.01302	170.01321	170.01295	170.01228	170.01230	170.01276
	B17	109.99913	109.99971	109.99936	109.99975	109.99994	109.99998	109.99916	109.99928	109.99958
	B15	29.99843	29.99875	29.99854	29.99885	29.99873	29.99892	29.99908	29.99920	29.99889
	B12	90.01109	90.01125	90.01098	90.01101	90.01134	90.01109	90.01106	90.01118	90.01111
	B10	160.01940	160.01953	160.01942	160.01913	160.01917	160.01899	160.01907	160.01911	160.01915
	B05	400.01278	400.01327	400.01314	400.01345	400.01312	400.01253	400.01256	400.01219	400.01283
	B03	450.01888	450.01941	450.01935	450.01992	450.02098	450.01996	450.02143	450.02035	450.02033
	B02	980.02721	980.02798	980.02736	980.02731	980.02840	980.02796	980.02750	980.02750	980.02767
	B01	1930.04077	1930.04298	1930.04046	1930.04071	1930.04197	1930.04236	1930.04239	1930.04187	1930.04163

## ME5000 Results

TABLE 6 (cont.)

From	To	LBL	FNAL	CEBAF	ANL	BNL	SLAC	SSC	SSC	Average
		357037	357046	357036	357086	357088	357089	357081	357075	
		1-92	1-92	1-92	2-92	3-92	6-92	8-92	8-92	
B12	B20	300.04598	300.04613	300.04625	300.04642	300.04661	300.04737	300.04717	300.04722	300.04684
	B19	260.02368	260.02373	260.02367	260.02393	260.02439	260.02399	260.02358	260.02414	260.02395
	B17	200.01034	200.01055	200.01045	200.01068	200.01118	200.01095	200.01056	200.01074	200.01076
	B15	120.00943	120.00972	120.00965	120.00987	120.01008	120.01023	120.01029	120.01039	120.01009
	B14	90.01107	90.01111	90.01104	90.01095	90.01100	90.01112	90.01095	90.01115	90.01104
	B10	70.00836	70.00850	70.00845	70.00779	70.00799	70.00809	70.00776	70.00811	70.00803
	B05	310.00197	310.00186	310.00209	310.00200	310.00164	310.00135	310.00160	310.00185	310.00176
	B03	360.00801	360.00794	360.00823	360.00865	360.00917	360.00921	360.00962	360.00918	360.00901
	B02	890.01681	890.01699	890.01626	890.01602	890.01652	890.01594	890.01661	890.01870	890.01668
	B01	1840.02997	1840.02881	1840.02943	1840.02920	1840.02987	1840.02970	1840.03016	1840.03138	1840.02996
B10	B20	370.05466	370.05486	370.05443	370.05458	370.05432	370.05515	370.05488	370.05483	370.05470
	B19	330.03189	330.03214	330.03174	330.03205	330.03220	330.03160	330.03126	330.03127	330.03169
	B17	270.01852	270.01891	270.01845	270.01869	270.01889	270.01915	270.01856	270.01827	270.01867
	B15	190.01774	190.01803	190.01771	190.01796	190.01780	190.01817	190.01833	190.01826	190.01804
	B14	160.01930	160.01953	160.01920	160.01913	160.01899	160.01908	160.01906	160.01893	160.01907
	B12	70.00831	70.00848	70.00830	70.00815	70.00782	70.00802	70.00737	70.00778	70.00791
	B05	239.99330	239.99390	239.99362	239.99373	239.99426	239.99321	239.99320	239.99384	239.99364
	B03	289.99941	290.00009	289.99981	290.00044	290.00164	290.00082	290.00092	290.00153	290.00086
	B02	820.00794	820.00847	820.00805	820.00792	820.00888	820.00914	820.00802	820.00938	820.00857
	B01	1770.02104	1770.02213	1770.02281	1770.02084	1770.02166	1770.02275	1770.02215	1770.02330	1770.02225
B05	B20	610.04849	610.04838	610.04787	610.04823	610.04828	610.04809	610.04883	610.04897	610.04838
	B19	570.02605	570.02584	570.02535	570.02569	570.02598	570.02567	570.02429	570.02466	570.02527
	B17	510.01239	510.01268	510.01198	510.01340	510.01248	510.01245	510.01194	510.01186	510.01235
	B15	430.01178	430.01189	430.01162	430.01240	430.01158	430.01120	430.01253	430.01191	430.01187
	B14	400.01323	400.01328	400.01286	400.01316	400.01261	400.01244	400.01220	400.01262	400.01265
	B12	310.00225	310.00205	310.00197	310.00223	310.00158	310.00136	310.00134	310.00108	310.00159
	B10	239.99385	239.99370	239.99357	239.99396	239.99378	239.99309	239.99328	239.99329	239.99350
	B03	50.00623	50.00619	50.00615	50.00668	50.00764	50.00775	50.00770	50.00777	50.00728
	B02	580.01498	580.01468	580.01447	580.01446	580.01503	580.01453	580.01520	580.01545	580.01486
	B01	1530.02963	1530.02860	1530.02846	1530.02691	1530.02862	1530.02859	1530.03046	1530.02979	1530.02881
B03	B20	660.05371	660.05456	660.05441	660.05581	660.05560	660.05621	660.05663	660.05706	660.05595
	B19	620.03132	620.03190	620.03154	620.03340	620.03328	620.03326	620.03257	620.03271	620.03279
	B17	560.01824	560.01843	560.01854	560.01992	560.01975	560.02032	560.02006	560.02041	560.01983
	B15	480.01750	480.01763	480.01778	480.01896	480.01871	480.01884	480.01971	480.01988	480.01898
	B14	450.01885	450.01905	450.01928	450.02027	450.02026	450.01989	450.02090	450.02040	450.02017
	B12	360.00785	360.00820	360.00823	360.00906	360.00919	360.00931	360.00946	360.00937	360.00910
	B10	289.99960	289.99987	289.99985	290.00077	290.00134	290.00096	290.00104	290.00143	290.00090
	B05	50.00605	50.00631	50.00621	50.00678	50.00721	50.00767	50.00752	50.00784	50.00721
	B02	530.00831	530.00855	530.00841	530.00762	530.00767	530.00784	530.00791	530.00761	530.00784
	B01	1480.02345	1480.02228	1480.02195	1480.02078	1480.02181	1480.02213	1480.02209	1480.02265	1480.02190
B02	B20	1190.06258	1190.06316	1190.06195	1190.06282	1190.06399		1190.06375	1190.06394	1190.06317
	B19	1150.03972	1150.04056	1150.03989	1150.04084	1150.04071		1150.03976	1150.03978	1150.04018
	B17	1090.02650	1090.02654	1090.02651	1090.02717	1090.02757		1090.02686	1090.02716	1090.02690
	B15	1010.02572	1010.02619	1010.02625	1010.02629	1010.02661		1010.02673	1010.02756	1010.02648
	B14	980.02743	980.02764	980.02807	980.02744	980.02750		980.02770	980.02770	980.02764
	B12	890.01651	890.01689	890.01676	890.01595	890.01637		890.01588	890.01641	890.01640
	B10	820.00815	820.00865	820.00855	820.00813	820.00905		820.00798	820.00908	820.00851
	B05	580.01433	580.01475	580.01466	580.01488	580.01493		580.01554	580.01545	580.01493
	B03	530.00808	530.00902	530.00835	530.00758	530.00733		530.00772	530.00790	530.00800
	B01	950.01299	950.01583	950.01339	950.01390	950.01405		950.01455	950.01488	950.01423
B01	B20	2140.07540	2140.07631	2140.07646	2140.07728	2140.07715		2140.07935	2140.07926	2140.07732
	B19	2100.05236	2100.05350	2100.05315	2100.05496	2100.05557		2100.05558	2100.05482	2100.05428
	B17	2040.03887	2040.03948	2040.03880	2040.04158	2040.04135		2040.04481	2040.04279	2040.04110
	B15	1960.03873	1960.03972	1960.03931	1960.04084	1960.03939		1960.04220	1960.04267	1960.04041
	B14	1930.04118	1930.04128	1930.04093	1930.04193	1930.04154		1930.04011	1930.04323	1930.04146
	B12	1840.02939	1840.03028	1840.02924	1840.03019	1840.02928		1840.03092	1840.03183	1840.03016
	B10	1770.02177	1770.02250	1770.02279	1770.02293	1770.02325		1770.02457	1770.02430	1770.02316
	B05	1530.02675	1530.02870	1530.02860	1530.02866	1530.02881		1530.03096	1530.03133	1530.02912
	B03	1480.02182	1480.02122	1480.02223	1480.02140	1480.02218		1480.02245	1480.02354	1480.02212
	B02	950.01355	950.01348	950.01330	950.01382	950.01502		950.01550	950.01488	950.01422
Sum		72002.541	72002.572	72002.554	72002.579	72002.613	45071.986	72002.618	72002.642	
ppm		-0.7	-0.2	-0.5	-0.1	0.3		0.4	0.7	

# ME5000 Results

TABLE 7. Difference between individual measurements and the average

From	To	Average	LBL 037 1-92	FNAL 046 1-92	CEBAF 036 1-92	ANL 086 2-92	BNL 088 3-92	SLAC 089 6-92	SSC 081 8-92	SSC 075 8-92
B20	B19	40.02289	-0.30	-0.29	-0.33	-0.64	-0.39	0.13	0.50	0.72
	B17	100.03602	-0.04	-0.10	-0.18	-0.22	-0.39	-0.16	0.28	0.69
	B15	180.03671	-0.11	0.23	-0.09	-0.18	-0.01	0.16	-0.06	0.16
	B14	210.03575	-0.37	-0.55	-0.71	-0.31	-0.04	0.11	0.50	0.47
	B12	300.04682	-0.28	-0.86	-0.65	-0.39	-0.23	0.25	0.58	0.44
	B10	370.05493	-0.30	-0.51	-0.43	-0.10	-0.22	0.20	0.03	0.53
	B05	610.04860	-1.19	-0.95	-0.60	0.39	-0.32	-0.06	0.16	0.45
	B03	660.05579	-2.73	-1.76	-1.79	-0.61	-0.34	0.67	0.73	1.33
	B02	1190.06317	-1.96	-0.91	-0.81	-0.73	-0.29	0.40	0.20	1.23
	B01	2140.07712	-2.33	-2.90	-0.95	-0.76	-1.84	-0.50	2.14	1.92
B19	B20	40.02278	-0.54	-0.15	-0.27	-0.42	-0.71	0.12	0.56	0.70
	B17	60.01319	0.12	0.22	0.11	0.25	-0.01	-0.16	-0.28	0.09
	B15	140.01393	0.28	0.34	0.22	0.43	0.56	0.03	-0.61	-0.63
	B14	170.01290	-0.20	-0.11	-0.32	0.30	0.51	0.11	-0.48	-0.13
	B12	260.02391	-0.20	-0.29	-0.33	0.03	0.57	0.12	-0.21	-0.19
	B10	330.03199	-0.03	0.04	-0.05	0.17	0.29	0.25	-0.27	-0.39
	B05	570.02559	-0.16	0.03	-0.04	0.85	0.38	-0.43	-0.56	-0.21
	B03	620.03250	-1.20	-0.76	-0.81	0.12	0.77	-0.50	0.05	0.36
	B02	1150.04024	-0.84	-0.34	-0.45	-0.26	-0.39	-0.12	-0.29	1.50
	B01	2100.05378	-1.92	-0.90	-0.87	-1.90	0.83	0.56	1.73	-0.35
B17	B20	100.03606	-0.38	-0.18	-0.32	-0.28	-0.46	-0.01	0.50	0.56
	B19	60.01310	0.10	0.36	0.15	0.28	-0.09	-0.18	-0.25	0.00
	B15	80.00066	0.05	0.28	0.17	0.12	0.64	0.37	-0.71	-0.59
	B14	109.99969	-0.41	-0.12	-0.34	0.29	0.50	0.41	-0.52	-0.36
	B12	200.01065	-0.34	-0.13	-0.29	-0.23	0.61	0.55	-0.31	-0.32
	B10	270.01874	-0.06	-0.01	-0.02	-0.16	0.35	0.35	-0.36	-0.15
	B05	510.01225	-0.07	0.18	-0.13	0.05	0.68	0.16	-0.49	-0.29
	B03	560.01935	-1.23	-0.69	-0.99	-0.45	0.90	0.52	0.12	-0.11
	B02	1090.02683	0.30	-0.32	-0.35	-1.19	1.38	0.03	0.29	-0.14
	B01	2040.04020	-0.95	-1.75	-1.53	-2.62	0.08	0.92	1.64	1.50
B15	B20	180.03683	0.20	0.03	-0.11	-0.17	-0.05	0.22	-0.04	0.16
	B19	140.01389	0.45	0.39	0.34	0.39	0.44	0.14	-0.77	-0.55
	B17	80.00084	-0.02	0.18	0.06	0.10	0.12	0.29	0.04	-0.62
	B14	29.99895	-0.57	-0.13	-0.48	-0.20	0.07	0.15	0.08	0.37
	B12	120.01010	-0.72	-0.30	-0.57	-0.36	0.14	0.13	0.14	0.50
	B10	190.01823	-0.44	-0.09	-0.20	-0.31	-0.01	-0.03	-0.07	0.61
	B05	430.01199	-0.65	-0.25	-0.31	0.13	0.52	-0.38	-0.56	0.61
	B03	480.01913	-1.70	-1.39	-1.31	-0.50	0.70	-0.09	0.06	1.12
	B02	1010.02689	-0.48	-0.73	-0.66	-0.93	0.65	0.20	-0.30	1.04
	B01	1960.04074	-0.73	-0.18	0.20	-1.60	1.44	-0.29	-1.06	1.32
B14	B20	210.03587	-0.59	-0.26	-0.55	-0.38	-0.15	0.04	0.85	0.16
	B19	170.01276	-0.08	0.18	0.01	0.27	0.45	0.19	-0.47	-0.45
	B17	109.99958	-0.45	0.13	-0.22	0.17	0.36	0.40	-0.42	-0.30
	B15	29.99889	-0.46	-0.14	-0.35	-0.04	-0.16	0.03	0.19	0.31
	B12	90.01111	-0.02	0.14	-0.13	-0.10	0.23	-0.02	-0.05	0.07
	B10	160.01915	0.25	0.38	0.27	-0.02	0.02	-0.16	-0.08	-0.04
	B05	400.01283	-0.05	0.44	0.31	0.62	0.29	-0.30	-0.27	-0.64
	B03	450.02033	-1.45	-0.92	-0.98	-0.41	0.65	-0.37	1.10	0.02
	B02	980.02767	-0.46	0.31	-0.31	-0.36	0.73	0.29	-0.17	-0.17
	B01	1930.04163	-0.86	1.35	-1.17	-0.92	0.34	0.73	0.76	0.24
B12	B20	300.04684	-0.86	-0.71	-0.59	-0.42	-0.23	0.53	0.33	0.38
	B19	260.02395	-0.27	-0.22	-0.28	-0.02	0.44	0.04	-0.37	0.19
	B17	200.01076	-0.42	-0.21	-0.31	-0.08	0.42	0.19	-0.20	-0.02
	B15	120.01009	-0.66	-0.37	-0.44	-0.22	-0.01	0.14	0.20	0.30
	B14	90.01104	0.04	0.08	0.01	-0.08	-0.03	0.09	-0.08	0.12
	B10	70.00803	0.33	0.47	0.42	-0.24	-0.04	0.06	-0.27	0.08
	B05	310.00176	0.21	0.11	0.34	0.25	-0.11	-0.41	-0.16	0.09
	B03	360.00901	-1.09	-1.07	-0.78	-0.36	0.16	0.20	0.61	0.17
	B02	890.01668	0.14	0.32	-0.41	-0.65	-0.15	-0.73	-0.06	2.03
	B01	1840.02996	0.01	-1.15	-0.53	-0.76	-0.09	-0.26	0.20	1.42

## ME5000 Results

Table 7 [cont.]

From	To	Average	LBL	FNAL	CEBAF	ANL	BNL	SLAC	SSC	SSC
			037	046	036	086	088	089	081	075
			1-92	1-92	1-92	2-92	3-92	6-92	8-92	8-92
B10	B20	370.05470	-0.04	0.16	-0.27	-0.12	-0.38	0.45	0.18	0.13
	B19	330.03169	0.20	0.45	0.05	0.36	0.51	-0.09	-0.43	-0.42
	B17	270.01867	-0.15	0.24	-0.22	0.02	0.22	0.48	-0.11	-0.40
	B15	190.01804	-0.30	-0.01	-0.33	-0.08	-0.24	0.13	0.29	0.22
	B14	160.01907	0.24	0.47	0.14	0.07	-0.07	0.02	-0.00	-0.13
	B12	70.00791	0.40	0.57	0.39	0.24	-0.09	0.11	-0.54	-0.13
	B05	239.99364	-0.34	0.26	-0.02	0.09	0.62	-0.43	-0.44	0.20
	B03	290.00086	-1.45	-0.77	-1.05	-0.42	0.78	-0.04	0.06	0.67
	B02	820.00857	-0.63	-0.15	-0.52	-0.65	0.31	0.57	-0.55	0.81
	B01	1770.02225	-1.21	-0.12	0.56	-1.41	-0.59	0.50	-0.10	1.05
B05	B20	610.04838	0.11	0.00	-0.51	-0.15	-0.18	-0.29	0.45	0.59
	B19	570.02527	0.78	0.57	0.08	0.42	0.71	0.40	-0.98	-0.61
	B17	510.01235	0.04	0.33	-0.37	1.05	0.13	0.18	-0.41	-0.49
	B15	430.01187	-0.09	0.02	-0.25	0.53	-0.29	-0.67	0.66	0.04
	B14	400.01265	0.58	0.63	0.21	0.51	-0.04	-0.21	-0.45	-0.03
	B12	310.00159	0.66	0.46	0.38	0.64	-0.01	-0.23	-0.25	-0.51
	B10	239.99350	0.35	0.20	0.07	0.46	0.28	-0.41	-0.22	-0.21
	B03	50.00728	-1.05	-1.09	-1.13	-0.60	0.36	0.47	0.42	0.49
	B02	580.01486	0.12	-0.18	-0.39	-0.40	0.17	-0.33	0.34	0.59
	B01	1530.02881	0.83	-0.20	-0.34	-1.89	-0.18	-0.22	1.66	0.99
B03	B20	660.05595	-2.24	-1.39	-1.54	-0.14	-0.35	0.26	0.68	1.11
	B19	620.03279	-1.47	-0.89	-1.25	0.61	0.49	0.47	-0.22	-0.08
	B17	560.01983	-1.59	-1.40	-1.29	0.09	-0.08	0.49	0.23	0.58
	B15	480.01898	-1.48	-1.35	-1.20	-0.02	-0.27	-0.14	0.73	0.90
	B14	450.02017	-1.32	-1.12	-0.89	0.10	0.09	-0.28	0.73	0.23
	B12	360.00910	-1.25	-0.90	-0.87	-0.04	0.09	0.21	0.36	0.27
	B10	290.00090	-1.30	-1.03	-1.05	-0.13	0.44	0.06	0.14	0.53
	B05	50.00721	-1.15	-0.89	-0.99	-0.42	0.01	0.46	0.32	0.64
	B02	530.00784	0.47	0.71	0.57	-0.22	-0.17	-0.00	0.07	-0.23
	B01	1480.02190	1.55	0.38	0.05	-1.12	-0.09	0.23	0.19	0.75
B02	B20	1190.06317	-0.59	-0.01	-1.22	-0.35	0.82		0.58	0.77
	B19	1150.04018	-0.46	0.38	-0.29	0.66	0.53		-0.42	-0.40
	B17	1090.02690	-0.40	-0.36	-0.39	0.27	0.67		-0.04	0.26
	B15	1010.02648	-0.76	-0.29	-0.23	-0.19	0.13		0.25	1.08
	B14	980.02764	-0.21	0.00	0.43	-0.20	-0.14		0.06	0.06
	B12	890.01640	0.11	0.49	0.36	-0.45	-0.03		-0.52	0.01
	B10	820.00851	-0.36	0.14	0.04	-0.38	0.54		-0.53	0.57
	B05	580.01493	-0.60	-0.18	-0.27	-0.05	-0.00		0.61	0.52
	B03	530.00800	0.08	1.02	0.35	-0.42	-0.67		-0.28	-0.17
	B01	950.01423	-1.24	1.60	-0.84	-0.33	-0.18		0.32	0.65
B01	B20	2140.07732	-1.92	-1.01	-0.86	-0.04	-0.17		2.03	1.94
	B19	2100.05428	-1.92	-0.78	-1.13	0.68	1.29		1.30	0.54
	B17	2040.04110	-2.23	-1.62	-2.30	0.48	0.25		3.71	1.69
	B15	1960.04041	-1.68	-0.69	-1.10	0.43	-1.02		1.79	2.26
	B14	1930.04146	-0.28	-0.18	-0.53	0.47	0.08		-1.35	1.77
	B12	1840.03016	-0.77	0.12	-0.92	0.03	-0.88		0.76	1.67
	B10	1770.02316	-1.39	-0.66	-0.37	-0.23	0.09		1.41	1.14
	B05	1530.02912	-2.37	-0.42	-0.52	-0.46	-0.31		1.84	2.21
	B03	1480.02212	-0.30	-0.90	0.11	-0.72	0.06		0.33	1.42
	B02	950.01422	-0.67	-0.74	-0.92	-0.40	0.80		1.28	0.66
		Max.	1.55	1.60	0.57	1.05	1.44	0.92	3.71	2.26
		Min.	-2.73	-2.90	-2.30	-2.62	-1.84	-0.73	-1.35	-0.64

### 2.3. Variance Component Analysis

As described in the previous paper (Horst Friedsam, Variance Component Analysis of Baseline Measurements), Koch's method of variance components<sup>1</sup> is used for the least-squares adjustment of the baseline measurements. The unknowns for which this method solves are the distances from the first pillar to each other pillar, the addition constant, and a two-part accuracy statement for the measurements.

Table 8 shows the VARCOM output for the CEBAF back set of measurements. Reference will be made to this sample in the ensuing description.

TABLE 8. VARCOM output

```

DATE : 06-19-1992          INPUT-FILENAME : CEBAF
TIME : 07:33              FILETYPE : BCK

PROGRAM VARCOM

ME5000 - CEBAF BASELINE MEASUREMENTS FROM B01 TO B20 B01=1 B20=11

VARIANZCOMPONENT ESTIMATION
=====

COMPUTATION WITH ADDITIONCONSTANT
-----

NUMBER OF BASELINE POINTS = 11

ALPHA1 = 1.00000 ALPHA2 = 1.00000 H = 1.00

M(s)**2 = SIGMA1**2 + SIGMA2**2 * S**(2*H)

*****

RESULTS AFTER 8 ITERATIONS

M0 = 1.000

SIGMA1**2 = 0.003 (mm)      M(SIG1**2) = 0.002 (mm)
SIGMA2**2 = 0.129 (mm/Km)  M(SIG2**2) = 0.037 (mm/Km)

FROM TO      S0 (m)      dS (mm)      S0 + dS      R.M.S. (mm)
=====
1 2 950.01330 0.091 950.01339 0.198
1 3 1480.02223 -0.415 1480.02181 0.191
1 4 1530.02860 -0.559 1530.02804 0.192
1 5 1770.02279 -1.098 1770.02169 0.191
1 6 1840.02924 0.651 1840.02989 0.192
1 7 1930.04093 -0.045 1930.04089 0.194
1 8 1960.03931 0.102 1960.03941 0.196
1 9 2040.03880 1.470 2040.04027 0.198
1 10 2100.05315 0.389 2100.05354 0.201
1 11 2140.07646 -0.414 2140.07605 0.206

ADDITION CONSTANT C = -0.032 (mm) M(c) = 0.027

```

... continued ...

<sup>1</sup>Fröhlich, H., Varianzkomponentenschätzung für elektrooptische Distanzmessungen auf Eichlinien, AVN, 89, pp. 411-417, 1982.

TABLE 8 [cont.]

FROM	TO	DIST(m)	RESIDUALS (mm)	M(s) (mm)	1/(M(s)**2)
1	2	950.01330	0.122	0.346	8.344
1	3	1480.02223	-0.384	0.535	3.494
1	4	1530.02860	-0.527	0.553	3.272
1	5	1770.02279	-1.066	0.639	2.451
1	6	1840.02924	0.682	0.664	2.270
1	7	1930.04093	-0.013	0.696	2.064
1	8	1960.03931	0.133	0.707	2.002
1	9	2040.03880	1.501	0.735	1.849
1	10	2100.05315	0.420	0.757	1.745
1	11	2140.07646	-0.382	0.771	1.681
2	3	530.00835	0.106	0.199	25.290
2	4	580.01466	0.022	0.216	21.408
2	5	820.00855	-0.217	0.300	11.097
2	6	890.01676	-0.228	0.325	9.471
2	7	980.02807	-0.544	0.357	7.854
2	8	1010.02625	-0.197	0.367	7.405
2	9	1090.02651	0.401	0.396	6.380
2	10	1150.03989	0.290	0.417	5.744
2	11	1190.06195	0.737	0.432	5.370
3	4	50.00621	0.048	0.060	279.605
3	5	289.99985	0.059	0.119	70.834
3	6	360.00823	-0.122	0.141	50.010
3	7	450.01928	-0.178	0.172	33.997
3	8	480.01778	-0.151	0.182	30.286
3	9	560.01854	-0.053	0.209	22.848
3	10	620.03154	0.216	0.230	18.898
3	11	660.05441	-0.147	0.244	16.797
4	5	239.99357	0.112	0.103	93.512
4	6	310.00197	-0.089	0.125	63.825
4	7	400.01286	0.015	0.155	41.800
4	8	430.01162	-0.218	0.165	36.846
4	9	510.01198	0.280	0.192	27.133
4	10	570.02535	0.179	0.213	22.110
4	11	610.04787	0.167	0.227	19.482
5	6	70.00830	-0.070	0.062	257.296
5	7	160.01920	0.025	0.081	152.410
5	8	190.01771	0.042	0.089	126.299
5	9	270.01845	0.159	0.113	78.914
5	10	330.03174	0.139	0.132	57.724
5	11	370.05443	-0.044	0.145	47.748
6	7	90.01104	-0.014	0.066	232.555
6	8	120.00965	-0.097	0.072	195.544
6	9	200.01045	-0.039	0.092	118.748
6	10	260.02367	0.010	0.109	83.419
6	11	300.04625	-0.063	0.122	67.189
7	8	29.99854	0.018	0.058	296.763
7	9	109.99936	0.056	0.069	207.619
7	10	170.01277	-0.085	0.084	143.116
7	11	210.03532	-0.127	0.095	111.704
8	9	80.00090	-0.010	0.064	245.085
8	10	140.01423	-0.071	0.076	172.834
8	11	180.03672	-0.054	0.086	134.398
9	10	60.01325	0.051	0.061	268.911
9	11	100.03574	0.069	0.067	219.965
10	11	40.02251	0.029	0.059	288.987

### 2.3.1. Baseline Distances

The distance unknowns determined by VARCOM are the distances from the first pillar to each other pillar. For a forward set of measurements these are the distances from pillar B20; for a back set they are the distances from B01. The fifth column of the table near the bottom of the first page of the sample output above gives the solutions to these unknowns.

A careful study of Tables 6 and 7 suggests a systematic trend: an increase over time in the distances between pillars. This trend is confirmed by the VARCOM results: Table 9 shows the results of the seven full sets of back measurements. Figure 4 shows graphically the proportional change (in parts per million) in each distance. The seven sets of measurements span seven months from January to August, from mid-winter to mid-summer. It is not clear whether this systematic trend is due to some systematic error in modelling the atmosphere or is instead indicative of a genuine physical change in dimension, perhaps due to the ground drying out during the long, dry Californian summer. What is clear is that the results are consistent to within a few tenths of 1 ppm, another indication that the ME5000 is genuinely operating within the claimed accuracy of  $0.2 \text{ mm} + 0.2 \text{ ppm}$ .

Table 9. A comparison of reduced distances from pillar B01

Lab. s/n	LBL 357037	FNAL 357046	CEBAF 357036	ANL 357086	BNL 357088	SSC 357081	SSC 357075	
Date	01-92	01-92	01-92	02-92	03-92	08-92	08-92	Mean
B02	950.01321	950.01324	950.01339	950.01406	950.01429	950.01542	950.01525	950.01448
B03	1480.02154	1480.02185	1480.02181	1480.02166	1480.02180	1480.02279	1480.02282	1480.02218
B05	1530.02730	1530.02795	1530.02804	1530.02844	1530.02918	1530.03065	1530.03080	1530.02942
B10	1770.02118	1770.02155	1770.02169	1770.02251	1770.02301	1770.02422	1770.02432	1770.02315
B12	1840.02948	1840.02991	1840.02989	1840.03065	1840.03082	1840.03209	1840.03206	1840.03110
B14	1930.04049	1930.04083	1930.04089	1930.04162	1930.04195	1930.04331	1930.04335	1930.04222
B15	1960.03887	1960.03944	1960.03941	1960.04045	1960.04082	1960.04265	1960.04255	1960.04118
B17	2040.03975	2040.04031	2040.04027	2040.04134	2040.04195	2040.04286	2040.04276	2040.04184
B19	2100.05312	2100.05359	2100.05354	2100.05470	2100.05523	2100.05587	2100.05587	2100.05504
B20	2140.07558	2140.07611	2140.07605	2140.07706	2140.07756	2140.07964	2140.07945	2140.07795
Sum	17740.3605	17740.3648	17740.3650	17740.3725	17740.3766	17740.3895	17740.3892	17740.3786

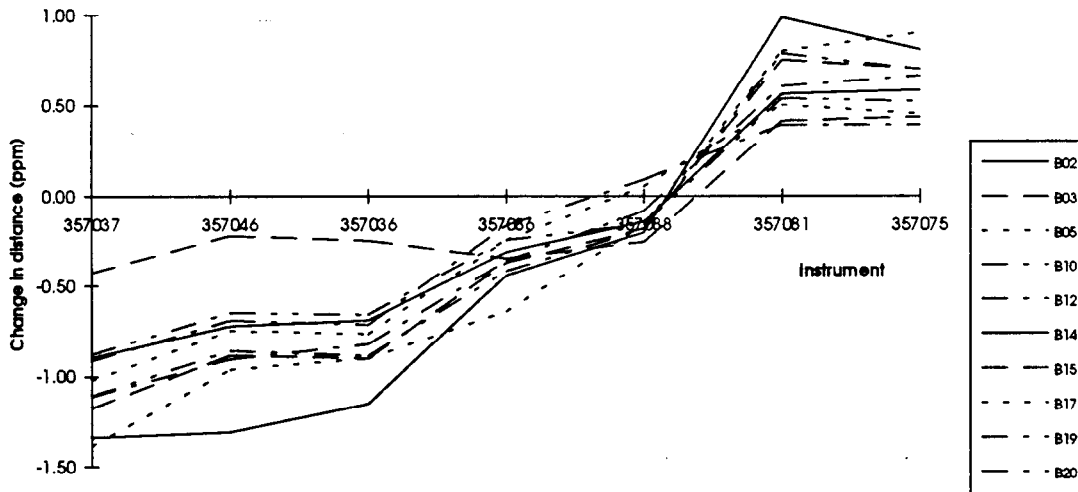


Figure 4. Proportional change in reduced distances from pillar B01

### 2.3.2. Addition Constant and Accuracy Statement

The VARCOM accuracy statement is given in two parts: a constant error  $\sigma_K$  and a proportional error  $\sigma_S$ ,

$$\sigma^2 = \sigma_K^2 + \sigma_S^2 S^{2H} \quad (1)$$

$S$  is the distance, and the value of  $H$  is chosen according to the relationship between accuracy and distance. For all full sets of baseline measurements (55 measurements between 11 pillars over 2140 m) it was found that  $H = 1$ . Kern's accuracy statement for the ME5000 is expressed in a similar form,

$$\sigma = 0.2 \text{ mm} + 0.2 \text{ mm/km} \quad (2)$$

The VARCOM program also provides an estimate of the accuracy of the determination of the three unknowns ( $\sigma_K$ ,  $\sigma_S$ , and the addition constant  $c$ ). A large error statement in comparison to the result itself indicates the result is insignificant. In the sample output above (Table 8), the standard errors of the determination of the unknowns show that  $\sigma_K$  and the addition constant  $c$  are insignificant. The only significant variable is the distance-dependant part of the accuracy statement,  $\sigma_K = 0.129 \text{ mm/km}$ .

Table 10 shows the results for the eight instruments measured at SLAC in 1992. It should be noted that the measurements with the SLAC instrument have not yet been completed. Insignificant results are indicated with asterisks.

TABLE 10. Results of the 1992 SLAC baseline measurements

	ME5000	Prism	Date	Direction	Accuracy		Add. const. $c$ mm
					$\sigma_K$ mm	$\sigma_S$ mm/km	
LBL	357037	365618	1-92	FWD (B20-B01)	***	0.337	***
				BCK (B01-B20)	0.022	0.064	***
FNAL	357046	365619	1-92	FWD (B20-B01)	***	0.352	-0.236
				BCK (B01-B20)	***	0.103	-0.167
CEBAF	357036	365630	1-92	FWD (B20-B01)	***	0.099	-0.029
				BCK (B01-B20)	***	0.129	***
ANL	357086	374425	2-92	FWD (B20-B01)	0.025	0.224	***
				BCK (B01-B20)	***	0.192	***
BNL	357088	375651	3-92	FWD (B20-B01)	***	0.387	-0.158
				BCK (B01-B20)	0.014	0.137	+0.067
SLAC	357089	375624	6-92	FWD (B20-B02)	***	0.324	-0.069
				BCK (B03-B20)	***	0.297	***
SSC	357075	375609	8-92	FWD (B20-B01)	***	0.414	***
				BCK (B01-B20)	0.012	0.194	***
SSC	357081	375608	8-92	FWD (B20-B01)	***	0.450	+0.192
				BCK (B01-B20)	0.037	0.442	+0.216

Two observations are necessary concerning the results given in Table 10.

1. For four out of the eight instruments the distance-dependant accuracy term  $\sigma_S$  is significantly better for the back set than for the forward set. The explanation for this lies perhaps in the fact that the ten distance unknowns for the back set (distances from B01 totalling 17740 m) are much longer than the ten distance unknowns for the forward set (distances from B20 totalling 5800 m). If this is so, then we may suggest that the  $\sigma_S$  result for the forward set is less reliable since it is more heavily influenced by short distances in which  $\sigma_K$  is dominant.
2. The forward and back sets for 357088 (BNL) produced substantially different values for the addition constant  $c$ . While all other instruments were fitted with the standard Kern LMC0500 tribrach which is permanently mounted and offers both true forced centering and fixed-height

levelling, the BNL instrument was fitted with the Wild GDF21K tribrach with Kern baseplate, which offers neither fixed-height levelling nor fully-accurate forced centering. The discrepancy in the BNL results is attributed to this tribrach.

### 2.3.3. Taylor-Hobson reflectors

In addition to the standard Kern ME5000 reflectors a total of seven Taylor-Hobson reflectors were measured. These are retro-reflectors inserted in 3½-inch diameter (88.9 mm) spheres (the same size as a Taylor-Hobson sphere). These are most conveniently used with the CERN socket since that was designed with the Taylor-Hobson sphere specifically in mind (the distance between centers of a 3½-inch sphere placed on a CERN socket is exactly 0.070 m).

A subset of four pillars (B14, B15, B17, B19) was used for these 3½-inch reflectors. For such short distances (the longest distance is 180 m)  $\sigma_K$  overwhelms  $\sigma_S$ , so a feature was added to the VARCOM program to set  $\sigma_S = 0$ . The addition constants for the seven reflectors are given in Table 11, in which asterisks again indicate an insignificant result.

Table 11. Addition constants for the Taylor-Hobson reflectors

	ME5000	Prism	Date	Addition constant $c$ (mm)		
				B19-B14	B14-B19	Mean
LBL	357037	LBL-TH1	4-92	-0.735	-0.515	-0.625
		LBL-TH2	4-92	-0.750	-0.615	-0.682
ANL	357086	ANL-TH1	3-92	-1.565	-1.615	-1.590
		ANL-TH2	3-92	-1.415	-1.565	-1.490
		ANL-TH3	3-92	-1.670	-1.560	-1.615
SLAC	357089	SLC-TH1	8-92	***	***	0.000
		SLC-TH2	8-92	0.155	0.095	0.125

## 3. INTERFEROMETER MEASUREMENTS

Two series of tests were made against the laser interferometer in the Sector 10 calibration laboratory:

1. A series of measurements over 1-2 mm in steps of  $c.100 \mu\text{m}$ .
2. A series of measurements along at least 10 m of the bench in steps of 1 or 2 m.

### 3.1. Short-Step Test

As described in a previous paper in this volume (Bernard Bell, ME5000 Test Measurements), reflectors were mounted back-to-back so that one faced the interferometer and the other faced the ME5000. These reflectors were near the laser head and interferometer but 30 m from the ME5000. The carriage on which the reflectors were mounted was moved in steps of  $c.100 \mu\text{m}$  for 1-2mm.

The purpose of these measurements was to evaluate the resolution of the instrument - its ability to discriminate small movements in the reflector. All the results exceed Kern's accuracy specification of  $0.2 \text{ mm} + 0.2 \text{ ppm}$ , with the results for 357046 (FNAL) and 357075 (SSC) particularly impressive, Fig. 5.

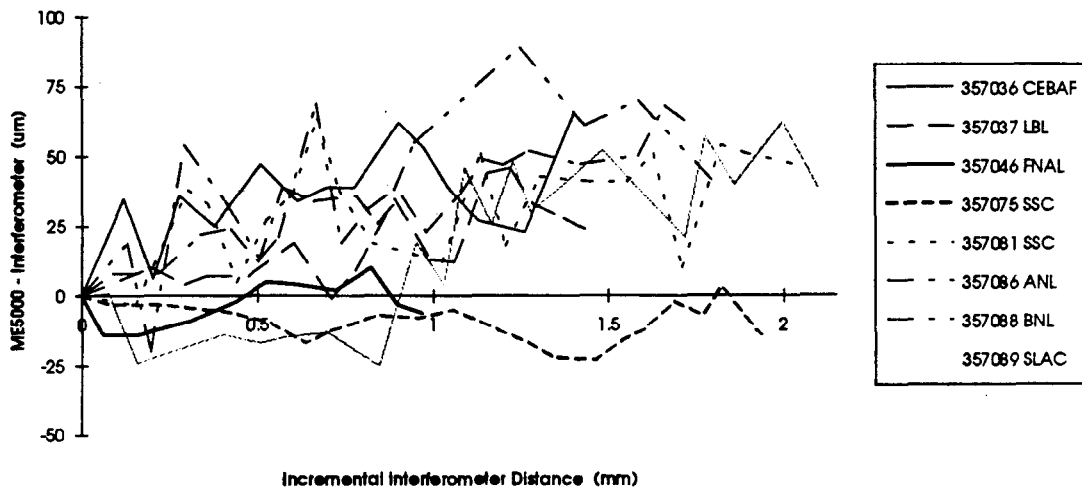


FIGURE 5. Short-step Interferometer Measurements

### 3.2. Long-step Test

The starting position for this series of measurements was the same as for the short-step test: the reflector was 30 m from the ME5000 and the interferometer was set to zero. In steps of 1-2 m the reflector was moved toward the instrument for a total of 10-15 m.

The purpose of this series of measurements was to evaluate the instrument's accuracy for short-range work. As for the short-step test, the results are all well within 0.2 mm with those for 357075 (SSC) and 357086 (ANL) particularly good, Fig. 6.

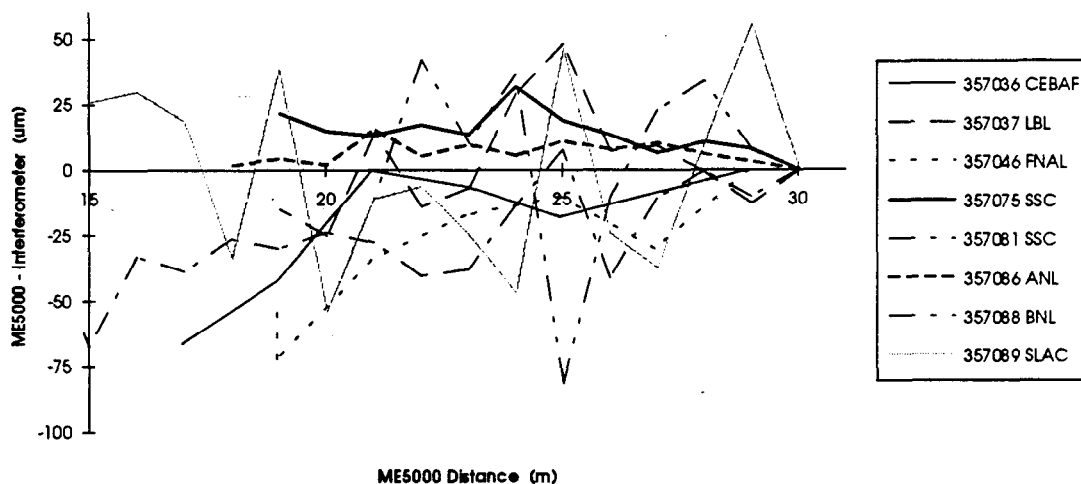


FIGURE 6. Long-step Interferometer Measurements

### 3.3. Conclusion

What is gained from these two types of measurements is more of a 'sense' of the short-range behavior of the ME5000 since neither type of measurement has any redundancy. Regrettably it has not been possible to evaluate short-range accuracy in a rigorous manner, *e.g.*, through trilateration of a small network or measurement of a closely-spaced series of collinear monuments.

The sense gained is that for short-range work all eight instruments operate well within an accuracy window of  $\pm 100 \mu\text{m}$ , given adequate meteorological correction, and that some instruments are very much better than this. Each instrument exhibits a distinct behavior for this short-range work, with some instruments especially good. For example, further measurements with 357075 (SSC) produced consistently remarkable results. Fig. 7 shows the results of a series of measurements in which the reflector was moved in ten steps of 20-30  $\mu\text{m}$  with the instrument 30 m from the reflector. Fig. 8 shows the results of a series in which the reflector was moved in 11 steps of 2 m, starting 29 m from the instrument and finishing 7 m from the instrument.

In order to appreciate the excellence of all these results it should be kept in mind that in all four charts (Figs. 5-8) the vertical scale is in  $\mu\text{m}$ .

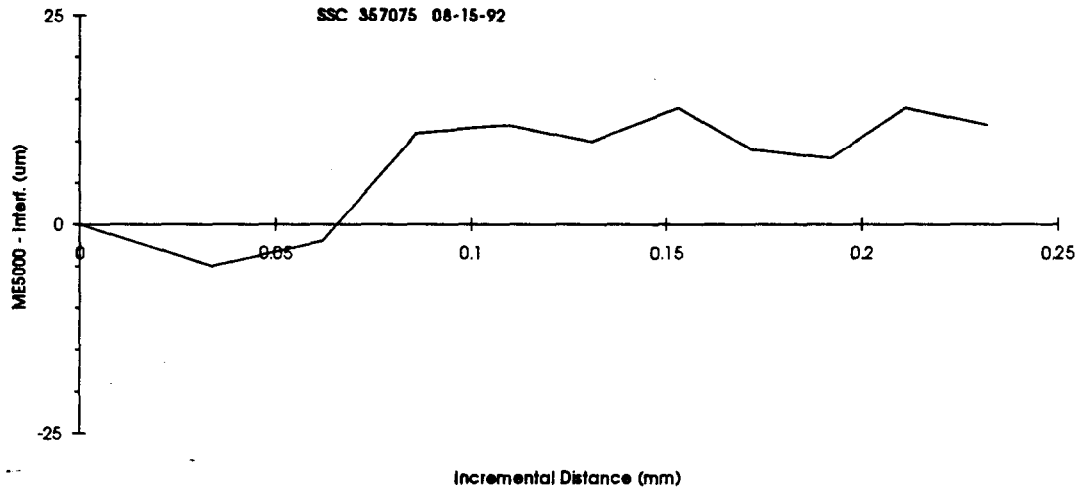


FIGURE 7. Interferometer comparison for instrument 357075 with step size of 20-30  $\mu\text{m}$ .

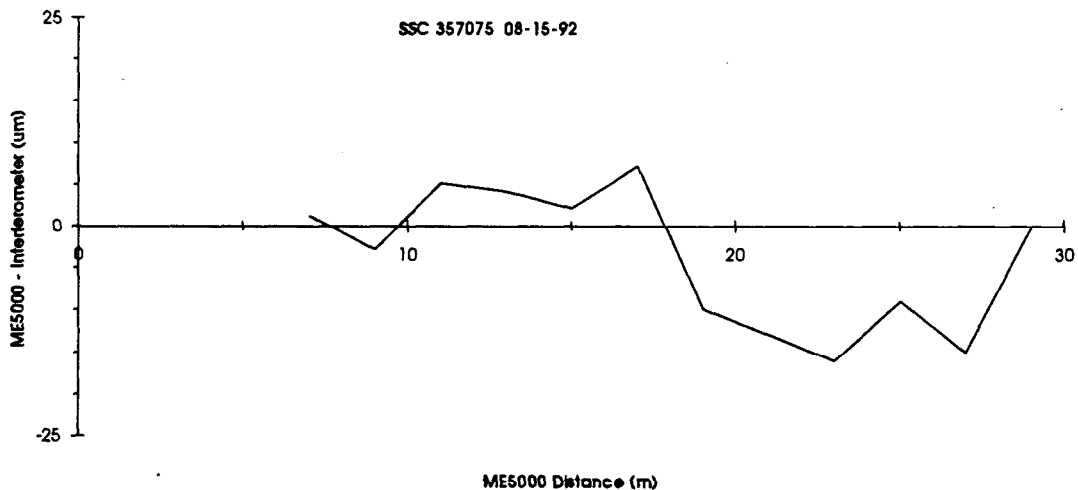


FIGURE 8. Interferometer comparison for instrument 357075 with step size of 2 m.

# **CALIBRATION AND USE OF THE MEKOMETER ME5000 IN THE SURVEY OF THE CHANNEL TUNNEL**

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## **ABSTRACT**

After briefly mentioning the background to the Eurotunnel project the paper describes the working experience of the Kern Mekometer ME5000. The use of the instrument on two geodetic networks is described, and the results of these measurements are discussed. Also discussed are the results of the regular calibrations on a seven pillar baseline. Conclusions are drawn as to the accuracy of the instrument under field conditions.

## **1. INTRODUCTION**

There have been plans for the construction of a tunnel beneath the English Channel since the eighteenth century. One of the first schemes was conceived by the French mining engineer Albert Mathieu in 1802 who proposed a tunnel for horse drawn carriages which was to be ventilated by chimneys projecting above sea-level. Digging was actually begun on an 1880 project using a rotary tunnelling machine designed by British engineer F. E. B. Beaumont. British fears of an invasion, however, halted work in 1882. Further projects followed, but it was in 1974 that construction work began on a three tunnel scheme. After 0.8 km of advance on the British side, and 1.2 km on the French, work was again halted, this time for economic reasons.

Interest in a tunnel was revived in the 1980's, and in 1985 Eurotunnel and two rival consortia submitted proposals. The Eurotunnel scheme followed the 1974 project in proposing the construction of three parallel tunnels. The two larger of these (7.6 m diameter) were to carry the main railway lines, and the smaller Service Tunnel (4.8 m diameter) located between the other two would be used for maintenance and to carry services. In 1986 Eurotunnel was awarded the concession to operate the tunnel and Trans Manche Link (TML) was contracted to carry out the construction. The Service Tunnel reached breakthrough point on December 1st 1990 with the two Running Tunnels completed by June the following year. This provides the first fixed link between the U.K. and Europe in modern time and represents not only an engineering triumph, but also a considerable surveying success.

Before work could begin, the geodetic relationship between the UK and France had to be precisely defined. The work carried out for the 1974 project served as the basis for the survey grid linking the two countries. Additional stations were added to the earlier scheme, since it was considered too sparse, and the whole network was extended to

ensure greater consistency. Measurements carried out by the Ordnance Survey of Britain and the IGN of France led to a new grid which was based on the Hayford 1924 Spheroid. This was called *Channel Tunnel Grid 1986* (CTG 86) and was considered to have an accuracy of 3 ppm. This grid was improved by the addition of GPS observations the following year and gave rise to the *Reseau Tunnel sous la Manche 1987* (RTM 87) with an accuracy considered to be 1 ppm. This was used for the remainder of the project. The largest unknown in the geodetic relationship between the two countries was that of the level datums. The 1974 project used the work of Cartwright and Crease of the National Institute of Oceanography published in 1963. Their studies of the sea surface using measurements of sea temperature, salinity, currents, etc., determined the French datum to be higher by 0.442 m. New work was undertaken in 1988 using GPS receivers which resulted in this figure being revised to 0.300 m with a standard error of 0.080 m. This value was accepted as the basis of the Channel Tunnel datum *Nivellement Trans Manche 1988* (NTM 88).

When the project began, it was realized that a significant amount of network measurement and monitoring would be required. The main RTM87 grid, although densified from the 1974 scheme, was still not adequate to serve as control for the large amount of construction which was needed at the British Terminal site, an area of roughly 4 km<sup>2</sup>. Thus a new network needed to be established and monitored. It was determined that the networks, with most lines over 1 km in length, were ideally suited to measurement with the ME5000. For these reasons a ME5000 was purchased in early 1988, just before the main construction activity at the Terminal site began.

## **2. MEASUREMENT OF THE MAIN NETWORKS**

### **2.1. The Two Main Networks**

As part of a monitoring scheme carried out by the Central Survey (QC) team at TML, two survey networks were measured on an annual basis. The first of these was the local RTM network consisting of twelve monuments from the main scheme (Figure 1). This was initially measured to serve as a check on the values given to TML by the Ordnance Survey and IGN. The second network was main Terminal net which was established from three of the RTM stations and consisted of eleven additional stations located mostly around the perimeter of the Terminal works (Figure 2). The monuments of these networks were used to transfer control needed to guide the initial tunnel drives, as well as to lay out structures and calibrate the gyro-theodolite. Their accuracy was, therefore, crucial to the success of the project as a whole.

The monuments were situated in a variety of locations, from the edges of cliffs and tops of hills to fields and even ancient fortifications. As such, the survey lines making up the network were anything but uniform. Lines ran from high cliffs to sea level, ran close to the ground, crossed valleys, and one line was almost entirely over the sea. The coastal location led to additional variability, as witnessed by the frequent fogs which rolled in

from the sea. In all, the area comprised a series of micro-climates which caused the very real possibility of serious problems in modelling representative atmospheric conditions.

## **2.2. Internal Agreement of Mekometer Distances**

Network measurements for the last survey of the main RTM network were carried out during the day over a two week period. Of the twelve stations occupied during the survey, six were pillars equipped with forced centering 120° spiders, and six were bolts over which tripods were set. In general, five distance measurements were made at each set up, and both ends of the line were occupied, usually on different days. Wet and dry bulb temperatures were measured using Cassella aspirated hygrometers ( $\pm 0.5^\circ\text{C}$ ), and pressure was measured using a Thommen TX-21 barometer ( $\pm 1.2$  mb). Meteorological measurements were taken at both ends of the line before and after each set of five distances. Direction sets measured with a Kern E2 were combined with the distances in the final adjustment.

Table 1 shows the differences between forward and reverse measurements. The line H to C1 is the exception to the general trend, and shows a 26.8 mm spread between three sets of measurements. This admittedly is over a 7.2 km line, but it still represents a 3.7 ppm disagreement. Uniquely, however, this line is almost entirely over the sea, from the end of Folkestone Harbor to the cliffs above the tunnel workings. It is therefore likely that the two land-based meteorological measurements were not representative of the conditions that existed over the sea. On a line of this length, a variation of  $3^\circ\text{C}$  in temperature would cause a 22 mm error in distance. All other lines agree to better than 5.0 mm, and most agree to better than 1.0 mm. The proportional difference averages 0.74 ppm for all 26 lines or 0.53 ppm if two lines (the shortest line and H to C1) are excluded. Error ellipses for an adjustment holding two points fixed (PP190 and C2), are all below 3.0 mm with the exception of Dover Castle (DCAS in Table 2a).

A similar exercise using the measurements of the Terminal network gives very different results. This network was measured in the same way as the RTM net, but was carried out later in the year under mostly sunny conditions. Of the fourteen stations included in this network, ten were forced centering pillars, and only four were tripods. The comparison of forward and reverse measurements can be seen in Table 3. Of twenty-nine lines eight show differences in excess of 2 ppm, the worst case being E to TM1 with a 8.45 mm spread over 2899 m giving 2.91 ppm. The average proportional difference is 1.39 ppm, significantly higher than that for the RTM network. The size of this difference could easily be attributed to atmospheric influences where a 1 ppm error can be caused by  $1^\circ\text{C}$  variation in temperature or 3 mb in pressure. Furthermore, since the measuring equipment, technique, and personnel were all identical to those used on the RTM work, the implication is that only the inability to accurately model atmospheric conditions could account for the difference in results. In spite of this greater variability in distances, error ellipses are still mostly below 3.0 mm with respect to the fixed RTM points (Table 2b). On many occasions serious shimmer made observing directions difficult, and some direction sets had to be weighted down because of their poor quality. However, the error ellipses show that good results can be achieved even under less than ideal observing

conditions. Trilateration adjustments of both networks produced results similar to the full adjustment, indicating the predominance of the distance measurements.

Under field conditions, therefore, it is apparent that accuracies approaching 0.5 ppm can be achieved even over varied terrain. If conditions prove unfavorable, accuracies may be reduced to 3 ppm or worse.

### **2.3. Mekometer Distances over Three Surveys**

A comparison of network lines measured over three surveys can be seen in Table 4. In general, lines in the RTM network agree well over the three years. All lines, with the exception of those to Castle Hill (CHILL), agree to better than 10 mm and most to better than 2 mm. Based on this repeatability it is clear that Castle Hill has shown considerable instability. The network adjustment confirmed this and defined movements of 30 mm in the first year, and 20 mm in the second. The proportional differences for all lines (excluding the shortest and lines to CHILL) average +0.21 ppm between the last two surveys (SRV3 and SRV2), and average +0.80 ppm between the first and last surveys (SRV3 and SRV1). However, the mean difference between SRV3 and SRV2 is +0.20 mm and is made up of 8 positive and 8 negative differences, whereas the mean difference between SRV3 and SRV1 is +2.06 mm and consists of 18 positive and only 2 negative differences. This suggests a systematic change between the first and third surveys.

This systematic effect is repeated with the Terminal data (Table 5). In this case an average difference of +0.90 mm exists between SRV3 and SRV2, with 10 positive and 9 negative differences, but between SRV3 and SRV1 the average difference is +4.55 mm and 11 of the 13 differences are positive. The average proportional difference here increases to 1.71 ppm. It would appear, therefore, that a systematic shortening effect of between 0.80 and 1.71 ppm can be detected in the distance measurements taken over a three year period. The possibility of movement or systematic meteorological effects, however, may have influenced these results. Further evidence from measurements carried out under the more controlled conditions of a multi-pillar baseline would reinforce these findings.

## **3. BASELINE CALIBRATIONS**

### **3.1. The Nottingham Baseline and Measurement Technique**

During the three year period that the ME5000 was in use at TML, a total of four multi-pillar baseline calibrations were carried out. Since no facilities existed close to the tunnel site, it was decided that the baseline at Nottingham University, 300 km north, would be the most suitable. The Nottingham baseline was established in the late seventies and is located on level farmland at Bunny just outside Nottingham. It is made up of seven pillars equipped with the Kern locking mechanism to ensure forced centering. The base is designed to provide a range of distances from 42 to 818 meters.

The baseline measurements were carried out at intervals of about eight months. On each occasion all combinations of distances were measured. This meant that seven set-

ups were made with six distances measured at each one. The 21 possible distances were consequently measured in both directions resulting in a total of 42 lines measured in all. Between three and five measurements were made of each line, and meteorological conditions were recorded in the same way as for the networks. All measurements took place during the day and took a full day to complete.

Measurements were reduced for atmospheric effects and then reduced to the horizontal. The mean reduced distance for each line was then adjusted following the method outlined by H.R. Schwendener (3). This method allows the additive constant, its standard error and the standard error of a single measurement to be determined. For the first calibration the reduced observations were also adjusted using the software at Nottingham University. Both methods of adjustment were compared and agreed satisfactorily.

### **3.2. Calibration Results over Four Surveys**

In Table 6 the adjusted distances over four surveys are compared. All adjusted distances together with the total distance measured are shown. It is clear that the shortening trend noticed in the network measurements can again be seen. From June 1988 to June 1990 a shortening of 1.28 ppm has taken place. This compares to a mean shortening for the networks of 1.25 ppm.

Since the distances along the baseline are collinear, the possibility of movement being the cause of this effect is eliminated. Also, systematic error in the atmospheric correction is doubtful since equipment was calibrated prior to use. It is likely, therefore, that the cause of this systematic trend lies in the instrument itself, and that it is probably due to the aging of the modulation crystal. It was not possible to confirm this with a frequency measurement.

### **3.3. Results of Adjustment**

Given below are the results of the Schwendener analysis for the four surveys. From this it is clear that on each calibration, the manufacturers stated accuracy of 0.2 mm was achieved. For the first three calibrations this was easily achieved with the mean standard error at 0.074 mm. The final calibration produced a significantly higher standard error of 0.155 mm. Again the influence of meteorological conditions can explain the difference in performance. The first three calibrations were carried out under stable atmospheric conditions, but on the last occasion there were significant fluctuations. Temperatures ranged from 13°C to 21°C during a day of mixed sunshine and showers. This variability in meteorological conditions undoubtedly led to a deterioration in measurement accuracy.

	Jun 90	Nov 89	Feb 89	Jun 88
	mm	mm	mm	mm
Addv k	0.029	0.046	0.032	0.011
s.e. k	0.073	0.027	0.037	0.041
s.e. measr.	0.155	0.058	0.079	0.086

The additive constant determined in the last calibration is consistent with the previous measurements, being almost exactly the mean of the others. However, it is practically insignificant, being less than the standard error for its determination in all but one case.

The baseline calibrations of the ME5000 produced results consistent with the manufacturers stated accuracy on all occasions. Given that the first three calibrations took place under stable meteorological conditions, these results indicate that a significantly higher order of accuracy can be achieved when atmospheric influences are suitably modelled.

#### **4. CONCLUSION**

Experience in using the ME5000 on the English Channel Tunnel has shown that an accuracy approaching  $0.075 \text{ mm} \pm 0.5 \text{ ppm}$  can be achieved with daytime measurements and the recording of meteorological conditions at each end of the line. Under unfavorable conditions, this accuracy can be degraded to  $\pm 3 \text{ ppm}$ . During the three years that the instrument was used on the primary networks of the project, some evidence of crystal aging was found.

Repeated calibration on a multi-pillar baseline has shown that the internal accuracy of the instrument was always better than that stated by the manufacturer. Under moderately favorable conditions, the accuracy is significantly better.

One advantage of using such an instrument is that it allows high quality measurements to be achieved even under poor observing conditions. The ME5000 was the main instrument used in the establishment and monitoring of the primary survey networks for the Channel Tunnel project and, as such, contributed to the success of the project as a whole.

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4. Dodson A.H. and Fleming P. The Geomensor CR204 : Baseline Test Results, *Survey Review*, 29, 229, 1988.

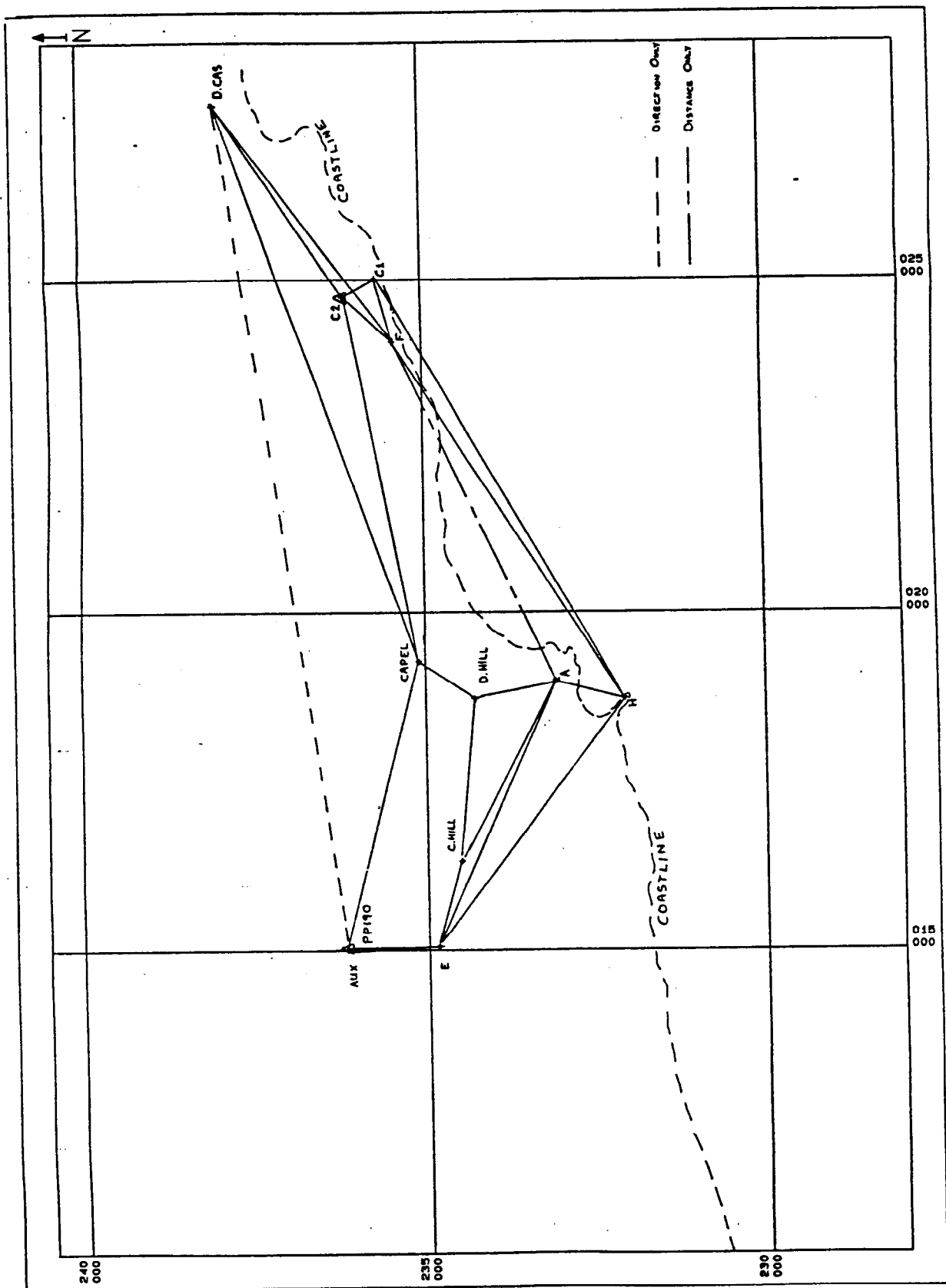


Figure 1. The RTM Network

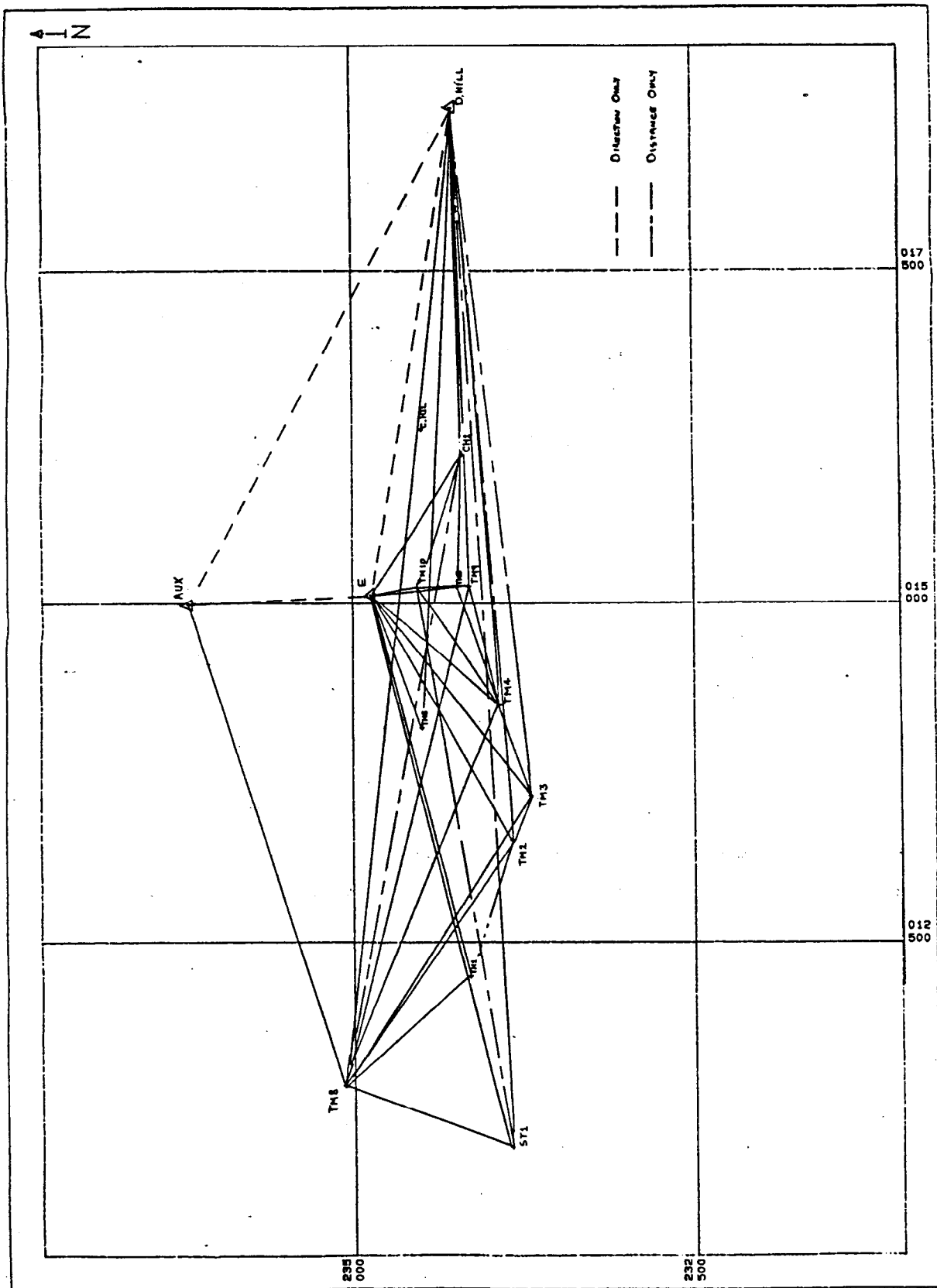


Figure 2. The Terminal Network

TABLE 1. Forward and Reverse Distances - RTM Net

	<u>STN</u>	<u>TARGET</u>	<u>DISTANCE</u> (m)	<u>DIFF</u> (mm)	<u>SPREAD</u> (mm) (ppm)	
1	AUX PP190	PP190 AUX	26.60621 26.60629	0.08	0.08	3.01
2	C1 C2	C2 C1	546.78270 546.78279	0.09	0.09	0.16
3	C1 F	F C1	955.35939 955.36154	2.15	2.15	2.25
4	C2 F	F C2	973.89102 973.89193	0.91	0.91	0.93
5	CAPEL DHILL	DHILL CAPEL	983.01671 983.01765	0.94	0.94	0.96
6	H A	A H	1041.03753 1041.03772	0.19	0.19	0.18
7	A DHILL	DHILL A	1207.03833 1207.03909	0.76	0.76	0.63
8	CHILL E	E CHILL	1306.40401 1306.40457	0.56	0.56	0.43
9	E PP190	PP190 E	1308.79047 1308.79096	0.49	0.49	0.37
10	E AUX	AUX E	1327.53980 1327.54038	0.58	0.58	0.44
11	DHILL H	H DHILL	2189.21166 2189.21211	0.45	0.45	0.21
12	DHILL CHILL	CHILL DHILL	2426.80308 2426.80364	0.56	0.56	0.23
13	CHILL A	A CHILL	3014.58804 3014.58891	0.87	0.87	0.29
14	C2 DCAS	DCAS C2	3385.71259 3385.71502	2.43	2.43	0.72

... continued ...

*Use of the ME5000 on the Channel Tunnel Project*

*TABLE 1 (cont.)*

	<u>STN</u>	<u>TARGET</u>	<u>DISTANCE</u> (m)	<u>DIFF</u> (mm)	<u>SPREAD</u> (mm)	<u>SPREAD</u> (ppm)
15	CHILL H	H CHILL	3409.93528 3409.93627	0.99	0.99	0.29
16	DHILL E	E DHILL	3718.01178 3718.01386	2.08	2.08	0.56
17	DHILL PP190	PP190 DHILL	4182.97243 4182.97336	0.93	0.93	0.22
18	DHILL AUX	AUX DHILL	4208.68626 4208.68767	1.41	1.41	0.34
19	A E	E A	4302.23767 4302.23965	1.98	1.98	0.46
20	F DCAS	DCAS F	4337.90709 4337.90727	0.18	0.18	0.04
21	CAPEL AUX	AUX CAPEL	4423.09266 4423.09703	4.37	4.37	0.99
22	E H	H E	4597.14665 4597.14888	2.23	2.23	0.49
23	CAPEL C2	C2 CAPEL	5567.83225 5567.83400	1.75	1.75	0.31
24	F A	A F	5612.34652 5612.35080	4.28	4.28	0.76
25	H F	F H	6323.71823 6323.72042	2.19	2.19	0.35
26	C1 H H	H C1 C1	7238.44809 7238.46432 7238.47485	16.23 10.53	26.76	3.70
MEAN :					<hr/> 0.74	

*Use of the ME5000 on the Channel Tunnel Project*

**TABLE 2. Error Ellipses**

**TABLE 2a: RTM Error Ellipses**

<u>STN</u>	<u>A (cm)</u>	<u>B (cm)</u>	<u>A/B</u>	<u>Phi (gon)</u>
PP190	0.00	0.00	0.0	100.00
AUX	0.02	0.01	2.4	-53.43
DCAS	0.94	0.13	7.5	-36.88
DHILL	0.19	0.12	1.6	20.20
H	0.19	0.16	1.2	45.65
E	0.14	0.05	2.5	95.15
F	0.24	0.06	4.2	-50.50
C2	0.00	0.00	0.0	100.00
C1	0.14	0.04	3.2	67.94
CHILL	0.27	0.13	2.1	22.17
A	0.19	0.14	1.3	27.33
CAPEL	0.23	0.14	1.7	-9.79

**TABLE 2b: Terminal Error Ellipses**

<u>STN</u>	<u>A (cm)</u>	<u>B (cm)</u>	<u>A/B</u>	<u>Phi (gon)</u>
AUX	0.00	0.00	0.0	100.00
DHILL	0.00	0.00	0.0	100.00
E	0.00	0.00	0.0	100.00
CH1	0.23	0.07	3.4	23.15
TM1	0.33	0.13	2.6	-9.13
TM2	0.26	0.09	2.8	-22.27
TM3	0.23	0.07	3.1	-32.12
TM4	0.18	0.06	2.8	-39.14
TM5	0.09	0.07	1.4	94.51
TM6	0.38	0.08	4.5	-19.46
TM8	0.35	0.11	3.1	0.42
TM9	0.09	0.05	1.9	94.11
TM10	0.08	0.04	1.9	88.00
ST1	0.42	0.16	2.6	-20.96

*Use of the ME5000 on the Channel Tunnel Project*

**TABLE 3. Forward and Reverse Distances - Terminal Net**

	<u>STN</u>	<u>TARGET</u>	<u>DISTANCE</u> (m)	<u>DIFFERENCE</u> (mm)	(mm)	<u>SPREAD</u> (ppm)
1	E TM10	TM10 E	330.42678 330.42680	0.02	0.02	0.06
2	TM3 TM2	TM2 TM3	355.72398 355.72466	0.68	0.68	1.91
3	TM10 TM9	TM9 TM10	378.77856 378.77858	0.02	0.02	0.05
4	TM3 TM4	TM4 TM3	701.60854 701.60893	0.39	0.39	0.56
5	E TM9	TM9 E	705.53420 705.53619	1.99	1.99	2.82
6	TM9 TM4	TM4 TM9	929.10354 929.10476	1.22	1.22	1.31
7	TM10 CH1	CH1 TM10	1049.21946 1049.22123	1.77	1.77	1.69
8	E TM6	TM6 E	1053.20746 1053.20822	0.76	0.76	0.72
9	TM10 TM4	TM4 TM10	1083.33812 1083.34107	2.95	2.95	2.72
10	TM1 TM8	TM8 TM1	1207.59721 1207.59768	0.47	0.47	0.39
11	E TM4	TM4 E	1244.89222 1244.89549	3.27	3.27	2.63
12	E CH1	CH1 E	1254.62435 1254.62498	0.63	0.63	0.50
13	TM8 ST1	ST1 TM8	1345.18541 1345.18641	1.00	1.00	0.74
14	E TM3	TM3 E	1886.87773 1886.87995	2.22	2.22	1.18
15	E TM2	TM2 E	2084.38957 2084.39228	2.71	2.71	1.30
16	TM3 TM8	TM8 TM3	2522.18141 2522.18424	2.83	2.83	1.12

... continued ...

*Use of the ME5000 on the Channel Tunnel Project*

**TABLE 3 (cont.)**

	<u>STN</u>	<u>TARGET</u>	<u>DISTANCE</u> (m)	<u>DIFFERENCE</u> (mm)	<u>SPREAD</u> (mm)	(ppm)
17	DHILL CH1	CH1 DHILL	2606.83250 2606.83657	4.07	4.07	1.56
18	E E TM1	TM1 TM1 E	2899.07957 2899.08636 2899.08802	6.79 1.66	8.45	2.91
19	TM4 TM8	TM8 TM4	3009.11769 3009.11795	0.26	0.26	0.09
20	DHILL DHILL TM9	TM9 TM9 DHILL	3593.01356 3593.01468 3593.02392	1.12 9.24	10.36	2.88
21	AUX TM8 AUX TM8	TM8 AUX TM8 AUX	3717.25393 3717.25876 3717.26082 3717.26529	4.83 2.06 4.47	11.36	3.06
22	TM8 TM9	TM9 TM8	3796.06354 3796.06426	0.72	0.72	0.19
23	E ST1 E	ST1 E ST1	4168.07860 4168.08687 4168.08920	8.27 2.33	10.6	2.54
24	DHILL DHILL TM4	TM4 TM4 DHILL	4503.20422 4503.20455 4503.21127	0.33 6.72	7.05	1.57
25	DHILL DHILL TM6	TM6 TM6 DHILL	4663.02175 4663.03017 4663.03160	8.42 1.43	9.85	2.11
26	DHILL TM3	TM3 DHILL	5184.62838 5184.62935	0.97	0.97	0.19
27	DHILL DHILL TM2	TM2 TM2 DHILL	5499.04133 5499.04146 5499.04777	0.13 6.31	6.44	1.17
28	DHILL DHILL TM8	TM8 TM8 DHILL	7324.36039 7324.36334 7324.37304	2.95 9.70	12.65	1.73
29	DHILL ST1	ST1 DHILL	7707.52011 7707.52541	5.30	5.30	0.69
MEAN :						1.39

*Use of the ME5000 on the Channel Tunnel Project*

**TABLE 4. RTM Distances over Three Surveys**

<u>STN</u>	<u>TARGET</u>	<u>SRV3</u>	<u>SRV2</u>	<u>SRV3-SRV2</u>		<u>SRV1</u>	<u>SRV3-SRV1</u>	
				mm	ppm		mm	ppm
PP190	AUX	26.60625	26.60597	-0.28	-10.52	26.60630	0.05	1.88
C2	C1	546.78274	546.78400	1.26	2.30	546.78455	1.81	3.31
F	C1	955.36046	955.35980	-0.66	-0.69	955.35850	-1.96	-2.05
F	C2	973.89147	973.89240	0.93	0.95	973.89140	-0.07	-0.07
DHILL	CAPEL	983.01718	983.01813	0.95	0.97	983.01800	0.82	0.83
A	H	1041.03762				1041.03930	1.68	1.61
DHILL	A	1207.03871				1207.03965	0.94	0.78
E	CHILL	1306.40428	1306.40393	-0.36	-0.28	1306.38440	-19.89	-15.22
PP190	E	1308.79071	1308.78890	-1.81	-1.38	1308.79265	1.94	1.48
AUX	E	1327.54009	1327.53877	-1.32	-0.99	1327.54175	1.66	1.25
H	DHILL	2189.21188	2189.21758	5.70	2.60	2189.21445	2.57	1.17
CHILL	DHILL	2426.80336	2426.80911	5.75	2.37	2426.81535	11.99	4.94
A	CHILL	3014.58847				3014.61850	30.03	9.96
H	CHILL	3409.93577	3409.95733	21.56	6.32	3409.98170	45.93	13.47
E	DHILL	3718.01282	3718.01019	-2.63	-0.71	3718.01650	3.68	0.99
PP190	DHILL	4182.97289	4182.96620	-6.69	-1.60	4182.97335	0.46	0.11
AUX	DHILL	4208.68696	4208.69146	4.50	1.07	4208.69035	3.39	0.81
E	A	4302.23866				4302.23890	0.24	0.06
AUX	CAPEL	4423.09484	4423.09460	-0.24	-0.05	4423.09845	3.61	0.82
H	E	4597.14776	4597.14920	1.44	0.31	4597.14790	0.14	0.03
C2	CAPEL	5567.83312	5567.83640	3.28	0.59	5567.83360	0.48	0.09
A	F	5612.34866				5612.35160	2.94	0.52
F	H	6323.71932	6323.71626	-3.06	-0.48	6323.72630	6.98	1.10
H	C1	7238.46240	7238.46378	1.38	0.19	7238.47220	9.80	1.35
MEAN(x CHILL) :				0.20	0.21	MEAN :	2.06	0.80

*Use of the ME5000 on the Channel Tunnel Project*

**TABLE 5. Terminal Distances over Three Surveys**

<u>STN</u>	<u>TARGET</u>	<u>SRV3</u>	<u>SRV2</u>	<u>SRV3-SRV2</u>		<u>SRV1</u>	<u>SRV3-SRV1</u>	
				mm	ppm		mm	ppm
TM3	TM2	355.72432	355.72471	0.39	1.10	355.72450	0.18	0.51
TM3	TM4	701.60874	701.60791	-0.82	-1.17	701.60645	-2.29	-3.26
TM9	TM4	929.10415	929.10715	2.99	3.22	0.00000	**	
E	TM6	1053.20784	1053.20811	0.27	0.26	1053.21000	2.16	2.05
TM4	E	1244.89386	1244.89363	-0.23	-0.18	1244.89740	3.55	2.85
CH1	E	1254.62467	1254.62608	1.41	1.12	1254.62910	4.44	3.54
TM8	ST1	1345.18591	1345.18898	3.07	2.28	0.00000	**	
E	TM3	1886.87884	1886.87664	-2.20	-1.17	1886.88445	5.61	2.97
TM2	E	2084.39093	0.00000	**		2084.39715	6.23	2.99
TM8	TM2	2172.65216	2172.64855	-3.61	-1.66	0.00000	**	
TM8	TM3	2522.18283	2522.18029	-2.53	-1.00	0.00000	**	
DHILL	CH1	2606.83454	2606.83406	-0.47	-0.18	2606.83925	4.71	1.81
E	TM1	2899.08465	2899.08278	-1.87	-0.65	2899.09435	9.70	3.35
TM8	AUX	3717.25970	3717.25759	-2.11	-0.57	0.00000	**	
TM8	TM9	3796.06390	3796.06093	-2.97	-0.78	0.00000	**	
ST1	E	4168.08489	4168.08968	4.79	1.15	4168.09535	10.46	2.51
DHILL	TM4	4503.20668	4503.21333	6.65	1.48	4503.21365	6.97	1.55
DHILL	TM6	4663.02784	4663.03265	4.81	1.03	4663.02750	-0.34	-0.07
DHILL	TM3	5184.62887	5184.63084	1.98	0.38	5184.63660	7.74	1.49
DHILL	TM8	7324.36559	7324.37320	7.61	1.04	0.00000	**	
MEAN :				0.90	0.30	MEAN :	4.55	1.71

*Use of the ME5000 on the Channel Tunnel Project*

**TABLE 6. Adjusted Baseline Distances over Four Surveys**

	<u>Jun-90</u>	<u>Nov-89</u>	<u>Change</u>	<u>Feb-89</u>	<u>Change</u>	<u>Jun-88</u>	<u>Change</u>
	(m)	(m)	(mm)	(m)	(mm)	(m)	(mm)
12	380.46384	380.46459	0.75	380.46441	0.57	380.46464	0.80
13	422.21516	422.21563	0.47	422.21555	0.39	422.21558	0.42
14	472.98475	472.98532	0.57	472.98533	0.58	472.98562	0.87
15	526.07598	526.07643	0.45	526.07659	0.61	526.07686	0.88
16	577.49541	577.49579	0.38	577.49594	0.53	577.49629	0.88
17	817.83593	817.83601	0.08	817.83617	0.24	817.83698	1.05
23	41.75132	41.75104	-0.28	41.75114	-0.18	41.75094	-0.38
24	92.52091	92.52074	-0.17	92.52092	0.01	92.52099	0.08
25	145.61214	145.61184	-0.30	145.61218	0.04	145.61222	0.08
26	197.03157	197.03120	-0.37	197.03154	-0.03	197.03166	0.09
27	437.37209	437.37142	-0.67	437.37176	-0.33	437.37234	0.25
34	50.76959	50.76969	0.19	50.76978	0.19	50.77005	0.46
35	103.86082	103.86080	-0.02	103.86104	0.22	103.86128	0.46
36	155.28026	155.28016	-0.10	155.28040	0.14	155.28072	0.46
37	395.62077	395.62038	-0.39	395.62062	-0.15	395.62140	0.63
45	53.09123	53.09110	-0.13	53.09126	0.03	53.09123	0.00
46	104.51066	104.51046	-0.20	104.51061	-0.05	104.51067	0.01
47	344.85117	344.85069	-0.48	344.85084	-0.33	344.85135	0.18
56	51.41944	51.41936	-0.08	51.41935	-0.09	51.41944	0.00
57	291.75995	291.75958	-0.37	291.75958	-0.37	291.76012	0.17
67	240.34051	240.34022	-0.29	240.34023	-0.28	240.34068	0.17
SUM :	5902.86350	5902.86245	-0.050	5902.86524	0.083	5902.87106	0.360
SF:	0.9999987	0.9999985		0.9999990			
ppm :	-1.281	-1.459		-0.986			